Math 240-002 Bob Powers

email rpowers@math.upenn.edu

office 4N73 DRL (4th floor end of long hall)

office hours 2:30-3:00 and after class

4 tests. Thurs Feb. 1st
Thurs Feb. 22d
Thurs March 22d
Thurs April 19th

One test grade replaced by the final (adjusted to account for the difference in test aveages) on the occasion that will help your score the most (if this does not help no replacement will be made.)

Final grade 60% test average (adjusted)

10% recitation

30% final

Distribution on final will determine grades (i.e. if 17 students get an A on the final about 17 A or A- will be given).

Book Differential Equations and Linear Algebra
First test Thurs Feb 4 will cover Chapters 2-4, 6.1,6.2

Matrices

Vector $X = (x_1, x_2, \cdots, x_n)$ $Y = (y_1, y_2, \cdots, y_n)$

$aX = (ax_1, ax_2, \cdots, ax_n)$ $X + Y = (x_1+y_1, x_2+y_2, \cdots, x_n+y_n)$

$aX + bY = (ax_1+by_1, ax_2+by_2, \cdots, ax_n+by_n)$

m x n matrix

m rows
n columbs

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix}$$

$AX = Y$

$$y_i = \sum_{j=1}^{n} a_{ij} x_j$$

Multiplication of an A (m x n)-matrix times B (n x p) matrix

$$C = AB$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

If A is an (m x n)-matrix and B is an (n x p) then B is an (m x p) matrix then B is a linear map of a p-dimensional vector space into a n-dimensional vector space and A is a linear map of an n-dimensional vector space into a m-dimensional space. So if we first apply B of a vector in p-space and then apply A to the resulting vector is n-space we get a vector in m-space. Since both A and B are linear maps the resulting map AB (note the matrices are written in reverse order) is a linear map of p-space into m-space which is an m x p matrix C. The formula for C is given by

$$C = AB$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

Example

Rotation around the z-axis by 90^o

$$R_z = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_z\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -y \\ x \\ z \end{bmatrix}$$

Rotation around the x-axis by 90^o

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad R_x\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -z \\ y \end{bmatrix}$$

$$R_X R_Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_Z R_X = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Notice $R_X R_Z \neq R_Z R_X$

The unit matrix or the identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A\overrightarrow{x} = \overrightarrow{x}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ identity matrix unit matrix

n x n unit matrix 1's on the diagonal and 0's off the diagonal.

$AI = A \qquad IB = B$

for I the identity matrix of appropriate size.

Important you must learn to multiply matrices.

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$