Here are some formulas for computing the exponential of a matrix.

If $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ the $e^{tA} = e^{at}\begin{bmatrix} \cos(bt) & -\sin(bt) \\ \sin(bt) & \cos(bt) \end{bmatrix}$

If A is a (2x2)-matrix with two distinct real eigenvalues $\{\lambda_1,\lambda_2\}$ then

$$e^{tA} = (\lambda_1-\lambda_2)^{-1}e^{\lambda_1 t}(A-\lambda_2 I) + (\lambda_2-\lambda_1)^{-1}e^{\lambda_2 t}(A-\lambda_1 I)$$

If A is a (2x2)-matrix with one real eigenvalue λ (λ is a repeated root of the characteristic equation $p(\lambda) = 0$) then

$$e^{tA} = e^{\lambda t}(I + t(A-\lambda I))$$

If A is a (3x3)-matrix with three real distinct eigenvalues $\{\lambda_1,\lambda_2,\lambda_3\}$

$$\begin{aligned} e^{tA} = {} & \frac{e^{\lambda_1 t}}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}(A^2-(\lambda_2+\lambda_3)A+\lambda_2\lambda_3 I) \\ & + \frac{e^{\lambda_2 t}}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}(A^2-(\lambda_1+\lambda_3)A+\lambda_1\lambda_3 I) \\ & + \frac{e^{\lambda_3 t}}{(\lambda_3-\lambda_2)(\lambda_3-\lambda_1)}(A^2-(\lambda_1+\lambda_2)A+\lambda_1\lambda_2 I) \end{aligned}$$

The above formula works for complex eigenvalues but is complicated.

If A is a (3x3)-matrix with a real eigenvalue λ_1 and a second repeated eigenvalue λ_2 (i.e. $p(\lambda) = -(\lambda-\lambda_1)(\lambda-\lambda_2)^2$) then

$$\begin{aligned} e^{tA} = {} & e^{\lambda_2 t}\, I + te^{\lambda_2 t}(A-\lambda_2 I) \\ & + (\lambda_2-\lambda_1)^{-2}(e^{\lambda_1 t} - e^{\lambda_2 t} + (\lambda_2-\lambda_1)te^{\lambda_2 t})(A-\lambda_2 I)^2 \end{aligned}$$

If A is a (3x3)-matrix with only one eigenvalue λ (so λ is a triple root of the characteristic equation $p(\lambda) = 0$) then

$$e^{tA} = e^{\lambda t}(I + t(A-\lambda I) + \tfrac{1}{2}t^2(A-\lambda I)^2)$$

If

$$A = \begin{bmatrix} \lambda & 1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & 0 & \cdots & 0 \\ 0 & 0 & \lambda & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda \end{bmatrix}$$

$$e^{tA} = e^{\lambda t} \begin{bmatrix} 1 & t & t^2/2! & t^3/3! & \cdots & 0 \\ 0 & 1 & t & t^2/3! & \cdots & 0 \\ 0 & 0 & 1 & t & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdot \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} B & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & D \end{bmatrix}$$

where B is an $(n_1 x n_1)$-matrix, C is an $(n_2 x n_2)$-matrix and D is an $(n_3 x n_3)$-matrix then

$$e^{tA} = \begin{bmatrix} e^{tB} & 0 & 0 \\ 0 & e^{tC} & 0 \\ 0 & 0 & e^{tD} \end{bmatrix}$$