Historical Background

The modern mathematical theory of probability came mainly from the Russian scholars P. L. Chebyshev (1821–1922), A. A. Markov (1856–1922), and Andrei Nikolaevich Kolmogorov (1903–1987). But the basic ideas arose much earlier, mostly in questions of games and gambling. In 1654, two French mathematicians, Pierre de Fermat (about 1601–1665) and Blaise Pascal (1623–1662), corresponded with each other regarding a problem posed by the Chevalier de Méré, a gambler and member of the aristocracy.

> *If the two players of a game are forced to quit before the game is finished, how should the pot be divided?*

Pascal and Fermat solved the problem by developing basic methods of determining each player's chance, or probability, of winning.

The Dutch mathematician and scientist Christiaan Huygens (1629–1695) wrote a formal treatise on probability. It appeared in 1657 and was based on the Pascal–Fermat correspondence.

One of the first to apply probability to matters other than gambling was the French mathematician Pierre Simon de Laplace (1749–1827), who is usually credited with being the "father" of probability theory.

Probability

If you go to a supermarket and select five pounds of peaches at 89¢ per pound, you can easily predict the amount you will be charged at the checkout counter.

$$5 \cdot \$0.89 = \$4.45.$$

This is an example of a **deterministic phenomenon.** It can be predicted exactly on the basis of obtainable information, namely, in this case, number of pounds and cost per pound.

On the other hand, consider the problem faced by the produce manager of the market, who must order peaches to have on hand each day without knowing exactly how many pounds customers will buy during the day. Customer demand is an example of a **random phenomenon.** It fluctuates in such a way that its value (on a given day) cannot be predicted exactly with obtainable information.

The study of probability is concerned with such random phenomena. Even though we cannot be certain whether a given result will occur, we often can obtain a good measure of its *likelihood,* or **probability.** This chapter discusses various ways of finding and using probabilities.

Any observation, or measurement, of a random phenomenon is an **experiment.** The possible results of the experiment are **outcomes,** and the set of all possible outcomes is the **sample space.**

Usually we are interested in some particular collection of the possible outcomes. Any such subset of the sample space is an **event.** Outcomes that belong to the event are "favorable outcomes," or "successes." Any time a success is observed, we say that the event has "occurred." The probability of an event, being a numerical measure of the event's likelihood, is determined in one of two ways, either *theoretically* (mathematically) or *empirically* (experimentally).

Theoretical Probability Formula

If all outcomes in a sample space S are equally likely, and E is an event within that sample space, then the **theoretical probability** of event E is given by the following formula.

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$$

$S = \{H, T\}$ E = heads

$P(E) = \frac{1}{2} = 0.5 = 50\%$

Flip a coin 3 times

$S = \{HHH, TTT, HTT, THT, TTH, THH, HTH, HHT\}$

$P(\text{at least } 2H\text{'s}) = \frac{4}{8} = 50\% = \frac{1}{2}$

2 or more

total # of outcomes

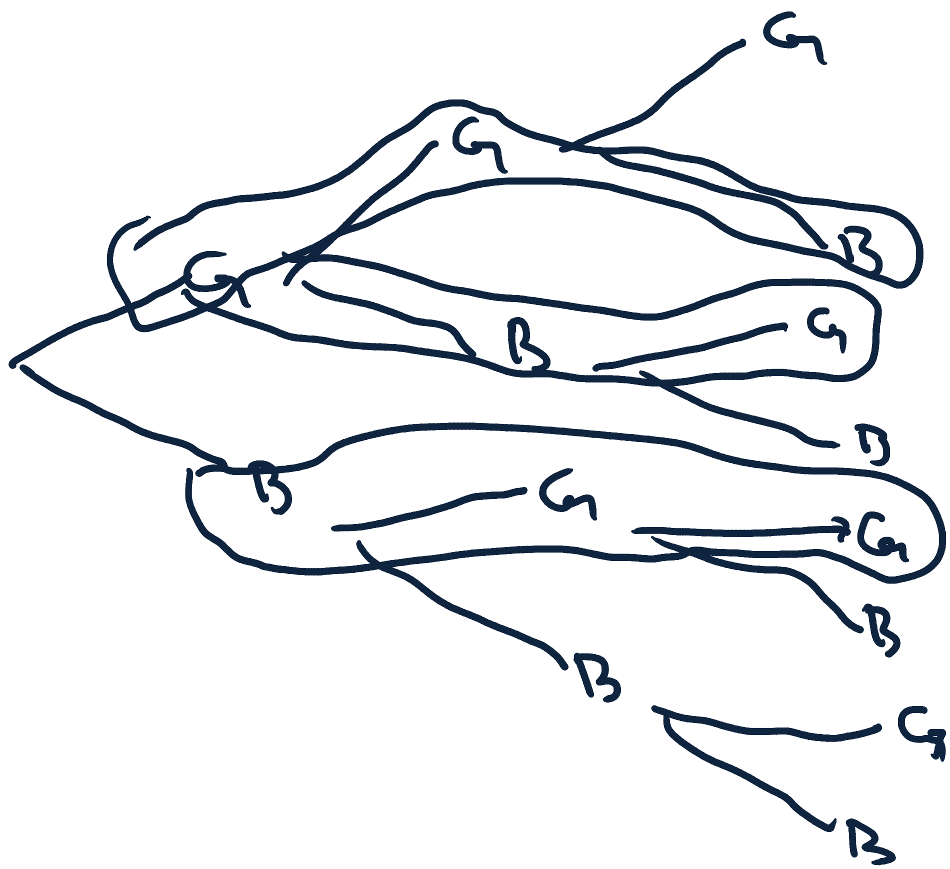
Kathy Campbell wants to have exactly two daughters. Assuming that boy and girl babies are equally likely, find her probability of success if

(a) she has a total of two children. **(b)** she has a total of three children.

a) $P(\text{exactly 2 Girls out of 2 children}) = \dfrac{1}{4} = 0.25 = 25\%$

$S = \{GG, BB, BG, GB\}$

b) $P(\text{exactly 2 G's out of 3 children}) = \dfrac{3}{8} = .375 = 37.5\%$



G
G
G
B
G
B
B
B
G
G
B
B
G
B

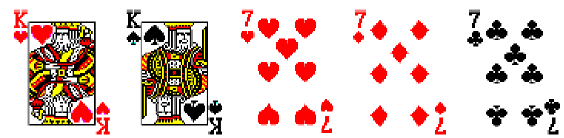
Find the probability of being dealt each of the following hands in five-card poker. Use a calculator to obtain answers to eight decimal places.

(a) a full house (three of one denomination and two of another)

Table 1 Number of Poker Hands in 5-Card Poker; Nothing Wild

Event E	Number of Outcomes Favorable to E
Royal flush	4
Straight flush	36
Four of a kind	624
Full house	3744
Flush	5108
Straight	10,200
Three of a kind	54,912
Two pairs	123,552
One pair	1,098,240
No pair	1,302,540
Total	2,598,960

Full House - Three cards of one rank and two cards of another rank



$$P(\text{full house}) = \frac{3744}{2{,}598{,}960}$$

3744/2598960
.001440576$\underbrace{2}$

0.144%

Empirical Probability Formula

If E is an event that may happen when an experiment is performed, then an **empirical probability** of event E is given by the following formula.

$$P(E) = \frac{\textbf{number of times event } E \textbf{ occurred}}{\textbf{number of times the experiment was performed}}$$

According to *Pocket World in Figures*, 2009 edition, published by *The Economist*, the U.S. population at the end of 2006 included 148.2 million males and 152.8 million females. If a person were selected randomly from the population in that year, what is the probability that the person would be a male?

$$P\left(\begin{matrix}\text{pick}\\ \text{a male}\end{matrix}\right) = \frac{148.2 \text{ mill}}{148.2+152.8 \text{ mill}}$$

```
148.2/(148.2+152
.8)
       .492358804
```

49.24%

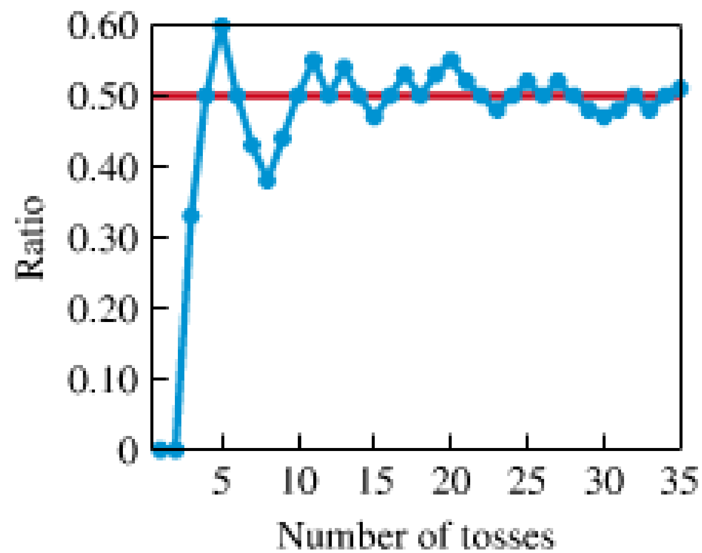
Law of Large Numbers

As an experiment is repeated more and more times, the proportion of outcomes favorable to any particular event will tend to come closer and closer to the theoretical probability of that event.

A fair coin was tossed 35 times, producing the following sequence of outcomes.

tthhh, ttthh, hthtt, hhthh, ttthh, thttt, hhthh

Calculate the ratio of heads to total tosses after the first toss, the second toss, and so on through all 35 tosses, and plot these ratios on a graph.



Ratio of heads to total tosses