Comparing Empirical and Theoretical Probabilities

A series of repeated experiments provides an *empirical probability* for an event, which, by *inductive reasoning*, is an *estimate* of the event's *theoretical probability*. (Increasing the number of repetitions increases the reliability of the estimate.)

Likewise, an established *theoretical probability* for an event enables us, by *deductive reasoning*, to *predict* the proportion of times the event will occur in a series of repeated experiments. (The prediction should be more accurate for larger numbers of repetitions.)

Odds

a+b
total # of outcomes

$\left\{ \begin{array}{l} a = \text{\# of favorable outcomes} \\ b = \text{\# of unfavorable outcomes} \end{array} \right.$

Whereas probability compares the number of favorable outcomes to the total number of outcomes, **odds** compare the number of favorable outcomes to the number of unfavorable outcomes. Odds are commonly quoted, rather than probabilities, in horse racing, lotteries, and most other gambling situations. And the odds quoted normally are odds "against" rather than odds "in favor."

$$P(E) = \frac{a}{a+b}$$

Odds

If all outcomes in a sample space are equally likely, a of them are favorable to the event E, and the remaining b outcomes are unfavorable to E, then the **odds in favor** of E are a to b, and the **odds against** E are b to a.

a to b $a:b$ $\frac{a}{b}$

$P(\text{Bob winning}) = \frac{12}{104}$

Bob Barickman has purchased 12 tickets for an office raffle in which the winner will receive an iPad. If 104 tickets were sold altogether and each has an equal chance of winning, what are the odds against Bob's winning the iPad?

fav $a = \boxed{12}$

unfav. $b = 104 - 12 = \boxed{92}$

odds in favor a to b 12 to 92

reduce $\boxed{3 \text{ to } 23}$

odds against b to a $\boxed{23 \text{ to } 3}$

Converting between Probability and Odds

Let E be an event.

1. ~~If $P(E) = \frac{a}{b}$, then the odds in favor of E are a to $(b - a)$.~~
2. If the odds in favor of E are a to b, then $\boxed{P(E) = \frac{a}{a+b}}$.

$$P(E) = \boxed{75\%} = \frac{75}{100} = \boxed{\frac{3}{4}} = \frac{a}{a+b}$$

$a = 3$

$b = 1$

Odds in favor

a to b

3 to 1

% to dec

$$0.144\% = 0.00144 = \frac{144}{100{,}000} = \frac{9}{6250}$$

move dec pt to left two spots

$$\frac{9}{6250} = \frac{a}{a+b}$$

$a = 9$

$b = 6241$

Odds in favor (Full house) $= \boxed{9 \text{ to } 6241}$

$$\frac{9}{6250} = \frac{1}{x} \Rightarrow \frac{9}{6250} = \frac{1}{694.4} = \frac{a}{a+b}$$

$a = 1$

$b = 693.\bar{4}$

$$\frac{9x}{9} = \frac{6250}{9}$$

$$x = 694.\bar{4}$$

Odds in favor of being dealt a full house $= 1$ to $\underbrace{693.\bar{4}}_{\text{can't have a fraction of a hand, so round up}}$

$\boxed{1 \text{ to } 694}$

If you dealt 694 hands, you should expect only one to be a full house

1 in 694

13. ***Number Sums for Rolling Two Dice*** The sample space for the rolling of two fair dice appeared in **Table 2** of **Section 10.1.** Reproduce that table, but replace each of the 36 equally likely ordered pairs with its corresponding sum (for the two dice). Then find the probability of rolling each sum.

$\boxed{\textbf{(a) } 2} = \frac{1}{36}$ **(b)** 3 $= \frac{2}{36}$ **(c)** 4 $= 3/36$

(d) 5 $= 4/36$ **(e)** 6 $= 5/36$ **(f)** 7 $= \frac{1}{6}$

(g) 8 $= 5/36$ **(h)** 9 $= 4/36$ **(i)** 10 $= \frac{3}{36}$

(j) 11 $= \frac{2}{36}$ **(k)** 12 $= \frac{1}{6}$

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\underline{6} \cdot \underline{6} = 36$

Roll your first set

$P\Big(\overbrace{\text{Rolling } \underbrace{7}_{6 \text{ ways}} \text{ OR Rolling } \underbrace{11}_{2 \text{ ways}}}^{\text{winning}}\Big) = \frac{8}{36}$

$P\left(\begin{matrix}\text{winning}\\ \text{on}\\ \text{1st Roll}\end{matrix}\right) = \frac{2}{9}$

$P\Big(\text{Roll } \underbrace{2}_{1 \text{ way}} \text{ OR Roll } \underbrace{3}_{2 \text{ ways}}\Big) = \frac{3}{36} = \frac{1}{12}$

Losing on 1st roll