We can add two vectors, what about multiplying two vectors?

Since adding two vectors yields another vector where the corresponding components are added, will the same work for multiplication?

No, the product of two vectors yielding another vector where the corresponding components are multiplied is meaningless.

There are actually two vector products that yield meaningful results but neither of these give a new vector using component-wise multiplication.

13.3 _____ Product

13.4 __________ Product

12.3 Dot Product

The **dot product** of $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is

$\mathbf{u} \cdot \mathbf{v} =$

Note: the result is a

Example:

$\mathbf{u} = \langle 2, -3, 4 \rangle$ and $\mathbf{v} = \langle 0, 6, 5 \rangle$

Properties of the dot product:

Let $\mathbf{u}, \mathbf{v},$ and $\mathbf{w}$ be vectors and let c be a scalar.

1. (Commutative Property)

2. (Distibutive Property)

3.

4.

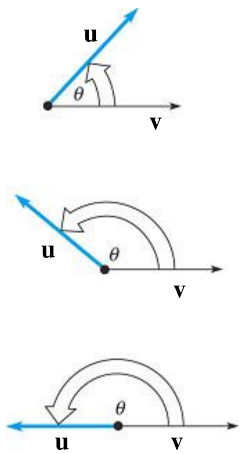
5.

Is the following expression meaningful?

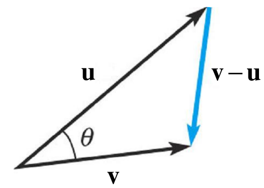
Math 114 – Rimmer
12.3 Dot Product

(a)	$(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$	**Yes**	**No**
(b)	$(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$	**Yes**	**No**
(c)	$\lvert\mathbf{u}\rvert(\mathbf{v} \cdot \mathbf{w})$	**Yes**	**No**
(d)	$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$	**Yes**	**No**
(e)	$\mathbf{u} \cdot \mathbf{v} + \mathbf{w}$	**Yes**	**No**
(f)	$\lvert\mathbf{u}\rvert \cdot (\mathbf{v} \cdot \mathbf{w})$	**Yes**	**No**

Find the angle between two vectors :



Let **u** and **v** be nonzero vectors, then



$$0 \le \theta \le \pi$$

Find the angle between two vectors :

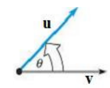
$\mathbf{u} = \langle 3, -1, 2 \rangle$ and $\mathbf{v} = \langle -4, 0, 2 \rangle$

Alternate form of dot product:

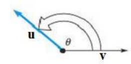
For nonzero **u** and **v**, $|\mathbf{u}| > 0$ and $|\mathbf{v}| > 0$

$\Rightarrow \mathbf{u} \cdot \mathbf{v}$ and $\cos\theta$ will always have the same sign

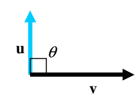
$0 < \theta < \frac{\pi}{2}$ $(\theta \text{ acute})$



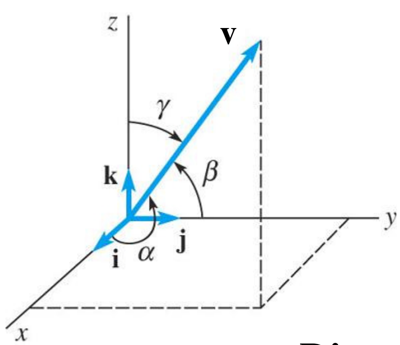
$\frac{\pi}{2} < \theta < \pi$ $(\theta \text{ obtuse})$



$\theta = \frac{\pi}{2}$ $(\theta \text{ right})$



Math 114 – Rimmer
12.3 Dot Product



Direction angles of **v** :

$\alpha =$ the angle between **v** and **i**

$\beta =$ the angle between **v** and **j**

$\gamma =$ the angle between **v** and **k**

Direction cosines of **v** :

$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$

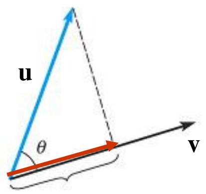
$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$

components of the unit vector in the direction of **v**

$\mathbf{i} = \langle 1, 0, 0 \rangle$

$\mathbf{j} = \langle 0, 1, 0 \rangle$

$\mathbf{k} = \langle 0, 0, 1 \rangle$



______________ of **u** onto **v**

has magnitude equal to :

the magnitude of the projection vector is called
the ______________ of **u** onto **v** or the _________ of **u** along **v**

There is an easier way to find this using $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$

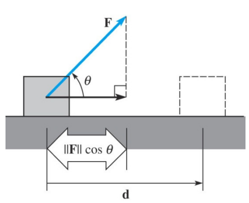
the vector projection of **u** onto **v** has $\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}$ as its magnitude and goes in the same direction as **v**

this can be done by taking the direction vector **v** and finding the unit vector in the same direction $\dfrac{\mathbf{v}}{|\mathbf{v}|}$

then scaling this unit vector by the desired magnitude $\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}$

When a constant force **F** moves an object a distance d in the same direction of the force,the work W done is :

If a constant force **F** applied to a body acts at an angle θ to the direction of motion, then the work done W is:



A toy wagon is pulled by exerting a force of 25 pounds on a handle that makes a 20° angle with the horizontal. Find the work done in Pulling the wagon 50 feet.