Math 104, Fall 2017 **Renato Ghini Bettiol**

Homework Set 7

DUE: NOV 13 - 15, 2017 (AT THE BEGINNING OF RECITATION)

1. Find the interval of convergence for each of the power series below.
   Do not forget to check the endpoints!

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^2 x^n}{2^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (x+3)^n}{\sqrt{n}}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(x-7)^n}{2^n}$

2. Find the Taylor Series of the following functions $f(x)$ centered at x_0:

   (a) $f(x) = x^3 - x^2 + x - 1$ at $x_0 = 0$

   (b) $f(x) = x^3 - x^2 + x - 1$ at $x_0 = 1$

   (c) $f(x) = \cos(2x^2)$ at $x_0 = 0$

   (d) $f(x) = \ln(3x+1)$ at $x_0 = 0$

3. Use differentiation term-by-term to find the Taylor Series of $f'(x)$ centered at x_0 for each of the items in the previous exercise.

4. The goal of this exercise is to derive the so-called *Leibniz formula* for π, namely

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \ldots.$$

   (a) Write the Maclaurin Series[1] of the function $f(x) = \dfrac{1}{1+x^2}$

   (b) Use integration term-by-term to find the Maclaurin Series of $F(x) = \arctan x$

   (c) Evaluate $F(1)$ using the series obtained in (b) to prove the Leibniz formula for π

[1] Recall the Maclaurin Series of $f(x)$ is simply the Taylor Series of $f(x)$ centered at $x_0 = 0$.