Here is a short list of possible projects for the presentations at the end of the term. Whether you choose from this list or not, you should consult with me and Léo about the suitability and the overall planning of your presentation.

You can work individually or in groups (2-3 students, depending on the complexity of the theme). Ideally, the length of presentation should be not more than: 30min for individual presentations, 50min for group of two, 60min for a group of three.

1) Construction of $\mathbb{R}$ via Dedekind cuts
   - Definition of Dedekind cuts, rational Dedekind cuts.
   - The semi-field $\mathbb{R}_0$ of non-negative Dedekind cuts, and $\mathbb{R}$ generated by $\mathbb{R}_0$.
   - Cauchy Completeness and the Completeness Axiom.

2) Continued Fractions
   - Definitions, convergence.
   - Rational numbers, periodicity.
   - The continued fractions of $e$ and $\pi$.
   - Best rational approximation of real numbers.

3) The Fundamental Theorem of Algebra
   - Statement of the Theorem.
   - Euler’s Theorem $e^{\pi i} = -1$ and De Moivre’s Formulas.
   - Proofs of the Fundamental Theorem of Algebra.

4) Gauss Integers and sums of two squares
   - Arithmetic in the ring $\mathbb{Z}[i] = \{ a + bi | a, b \in \mathbb{Z} \}$ (Euclidean algorithm)
   - Prime numbers of the form $p = k^2 + l^2$ with $k, l \in \mathbb{N}$.
   - Natural numbers of the form $n = k^2 + l^2$ with $k, l \in \mathbb{N}$.

5) Euler’s 4 Square Theorem
   - The skew field $\mathbb{H} = \{ a + bi + cj + dk | a, b, c, d \in \mathbb{R} \}$ of Hamiltonian quaternions.
   - The ring of integer quaternions $\mathbb{H}_\mathbb{Z} = \{ a + bi + cj + dk + \frac{e}{2} (1 + i + j + k) | a, b, c, d, e \in \mathbb{Z} \}$.
   - Arithmetic in $\mathbb{H}_\mathbb{Z}$ (Euclidean algorithm).
   - The 4 Square Theorem.

6) Transcendence of $\pi$ and $e$ (Lindemann–Weierstrass Thm)
   - Introduction to the question about Squaring the Circle.
   - Algebraic numbers numbers vs transcendental numbers.
   - The proof.

7) The Riemann Zeta Function
   - Definitions, Euler product.
   - $\Gamma$-function and the functional equation.
   - The Riemann Hypothesis.

8) Curves vs. Surfaces
   - Surfaces cannot be homeomorphic to curves.
   - Space-filling curves / Peano curve.
   - Continuous nowhere differentiable function.
   - Differentiable function which are not $C^1$. 