**Beta priors and posteriors.**

(a) Construct a model in which $p$ is a parameter picked according to a probability distribution $\mu$ on $[0, 1]$ and (given that) $\{X_n\}$ are IID Bernoulli variables with mean $p$. You should have a formal probability space on which $p$ and the $X_n$ are random variables with the correct properties.

(b) What is the conditional distribution $(p | X_1)$ of $p$ given $X_1$? This is called the **posterior** distribution of $p$ given the prior distribution $\mu$ and data $X_1$.

**NOTE:** This part is the heart of this exercise. The way to rigourously compute a conditional distribution is to guess and verify.

(c) What is the posterior after seeing $n$ bits of data, that is, what is $(p | X_1, \ldots, X_n)$?

(d) Specialize to $\mu \sim \beta(a, b)$, that is, a beta distribution with parameters $a$ and $b$. What is $(p | X_1)$? What is $P(X_2 = 1 | X_1 = 1)$?

(e) In the case $\mu \sim \beta(a, b)$ what is $(p | X_1, \ldots, X_n)$?

(f) What is $P(X_{n+1} = 1 | X_1, \ldots, X_n)$?