DISCLAIMER:

In training one's self or others to teach a Socratic/Cooperative style class (henceforth SC), it seems no amount of preparation or advice can substitute for a certain kind of on-the-job training. The essential ingredients of on-the-job training are criticism and imitation. Experienced instructors visit the classrooms of new instructors, taking extensive notes on what they see: what went right, what went wrong, what might have worked better, and so on. New instructors also visit the classrooms of experienced instructors, taking equally careful notes on what went well or poorly, what might have worked better, and on what techniques they see used that they would like to use themselves. Some of this type of work can be done beforehand, via visits in the previous semester or videotaped classes, but it tends to be more valuable when it comes after the new instructor has had a chance to try teaching a class or two first. Currently in 130-131-132 we are trying for one visit each direction in the first two weeks, another in the second two weeks, plus at least one more during the term.

If you are not going to adhere to a plan as outlined above, then **don't expect the notes that follow to do much for you**. You can't learn to play the piano by reading books about it or by talking about technique with Rubenstein, so don't try to learn a significant new teaching skill without practice and guidance either. On the plus side, all the interested instructors that have taught SC classes here (admittedly a self-selected sample) have become pretty good at it, so you can be pretty confident that you'll be able to step right in and teach effectively this way even if you're not a virtuoso.

1 Basics

1.1 The philosophy

Unless you've been on Mars, you're probably aware of the age-old battle between "skills" advocates and "process" advocates. I'm from the "process" camp but I hope to avoid

a lot of the partisanship that is prevalent in discussions of pedagogy and stick mainly to points both sides agree on. We want students to come away from (lower level) math classes with certain skills and attitudes. In particular we want them to reason and prove, to translate between words and symbols, to perform algebraic manipulations correctly and with understanding of the justification, and to solve problems other than clones of problems they have been shown how to do. Whether or not you believe specific skills and knowledge to be paramount (arithmetic of negative numbers, solution of quadratic equations, propositional logic, summation of common series), you undoubtedly want them to know these things in the ways mentioned above: verbally and symbolically, with justification and proof, well enough to apply to new situations.

The tenet underlying SC classes is that students need to learn that they can think for themselves, and that they will be able to learn properly if and only if they are forced or enticed to continue thinking things through on their own. Realize please that this does not apply to students who already have this skill. I don't think we need SC classes at the advanced undergraduate level, and they become increasingly inefficient at higher levels. If college admissions standards (or high school graduation standards) were what we'd like them to be, we wouldn't need SC classes in college at all. The students who benefit from SC classes are ones I would term remedial: those taking pre-calculus, and those in the Ed program who are required to take what is essentially junior high school mathematics.

Our philosophy with these students is to do anything we can to get them to think and speak mathematics, and then once we have them going, to exact from them a quality of mathematical reasoning that is higher than anything ever asked of them in a traditional course, thus ensuring that they learn the course content in a useful and permanent way. The meta-skill we emphasize is for the students to **know when they know something**, versus when they are just guessing or are confused. In the time-span of the course or sequence of courses we move from a "process is everything, choice of content matters little" approach to a stage where we cover the traditional content and expect students to focus on these topics and skills while applying the critical thinking they have learned in the first phase. The first phase is the harder phase for most instructors, since we have to be psychologists - and

sometimes mind-readers - in addition to being mathematicians. These notes concentrate on this phase, though they apply to the other as well.

Perhaps the most controversial part of this approach is our unwillingness to tell students the answer. Some skills can only be acquired by imitation, so it is possible to be overly dogmatic on this point. The basis for our approach is that much of this material is accessible to these students, with a little help from us, and that our habitually providing answers will cause their problem-solving ability to atrophy, though it may increase their rate of skill acquisition (I would argue that it won't). Thus we make every attempt when discussing an attempted solution in class not to tip off whether it is right or wrong until the whole class has had a chance to criticize it or register comprehension and agreement. Depending on the context, we do or do not in the end provide model solutions.

1.2 Typical classroom mechanics

A usual 50 minute class consists of two kinds of time: some time when the students are working in groups of 3 or 4 on a problem or worksheet (set of problems) and some time when the entire class is discussing the problem set. Some instructors enjoy keeping to a familiar rhythm, spending the initial 25 to 30 minutes each day working in small groups, then spending the latter part of the lesson in a large group discussion detailing what the various small groups found, where they got stuck, and so forth. Often there are parts of the worksheet that are not covered in this phase; some of these are assigned for homework and some are discarded. Other instructors prefer to go back and forth a little more, starting in a small group, then convening the large group when most small groups are done with the first problem, discussing it a bit, then remanding the class into small groups, and so on. When a class meets for 75 minutes twice a week instead of 50 minutes three times a week, it is usually necessary to go back and forth this way, and it is also often convenient to continue a large group discussion from the end of one class at the beginning of the next.

During the small group working time, the instructor circulates among the groups, an-

swering questions when necessary, doling out encouragement when necessary, challenging the students to justify what they claim to have figured out or to explain their half-baked ideas. Often the mere presence of the instructor encourages a renewed attack on a problem.

The large group discussion begins with the students explaining what they have done. Other students are required to listen carefully and to register agreement, disagreement or lack of comprehension. Once the explanation is comprehensible, those in disagreement are encouraged to justify their disagreement, with the aim of a resolution or synthesis. The instructor plays moderator as long as fruitful ideas are being produced, but slips into the role of leader when needed. For example, if no one challenges something wrong, or if there is a disagreement but it is too inarticulate to produce a good synthesis, then the instructor may rephrase what has been said so as to sharpen the contradiction or caricature a wrong approach, in a way that forces a light to dawn for at least some students.

1.3 Highly recommended procedures

Before getting into specifics of classroom technique, here are a few simple procedures that make a large difference.

- *Nametags.* Have the students wear nametags each day until you know their names (in my case 2 or 3 weeks). When calling on students, call on them by name and in general attempt to use their names frequently. This serves two purposes. First, by attempting to learn their names, you create a separate mental category for each student, which helps you pay attention to how each student is doing and to their individual needs. Secondly, there seems to be a psychological advantage to students hearing their names. Coming around to a small group and asking "Amy, can you tell me what progress your group has made?" or asking in a large group discussion whether "Sam's objection to Cindy's idea" holds water elicits more of a response than the same questions without the names identified. Somehow, students are more prone to take their own beliefs seriously when names are attached.

- *Randomize groups.* I usually assign groups using playing cards randomly dealt: all the aces for a group, the twos form another, and so on. I re-form groups twice or thrice during the semester. Preventing students from choosing their buddies for a group helps them form connections, subject their ideas to the intellectual marketplace, and treat all the others in their groups fairly. The playing cards themselves lend an air of intrigue as students await the results of the lottery.

- *Get enough sleep.* Alertness is required on the part of the instructor. You'd be surprised what a difference this makes. You can fake it when you're lecturing, but try playing moderator when you can't concentrate or respond quickly, and you'll see what I mean.

- *Start the semester with a bang.* That is, don't spend the first day on administrative stuff and the second on some kind of review. Jump into an absorbing problem on Day 1, preferably a tried-and-true chestnut, and fill in the administrative details later, when they've gotten the idea of what the class will be about. The tone of the first day's discussion sets an example that's hard to erase, so make sure it is as lively as possible.

- *Be yourself* at least up to a point. If you're the goofy type, be goofy, if you're serious and intense, let them feel the intensity, if you're understated and direct, be that way. You are on stage, and want to use your charisma, but don't pretend to be someone you're not.

2 Class composition and small group dynamics

2.1 Doing the rounds

When you first assign a problem to work on in small groups, there may not be much you have to do. No one is stuck yet; no one needs your help. There is a lot you can accomplish

in this time. First, you can quickly take attendance group by group - after you know the names this takes less than half a minute. After this, you will want to quickly "do the rounds". Visit each group once just to check that they have gotten down to work. On the first round, look for any trouble with the wording of the problem that may be holding people up. If it's part of their job to decipher it, encourage them to do so. If it's a mistake, or if you need to supply a definition, then make a quick announcement. On your second round you can linger longer. This is a good time to make a mental note of which groups are going faster than the others. It helps, during the subsequent large group discussion, to have a good idea of who has gotten how far. If a group has quickly and incorrectly or incompletely answered a problem and gone on to another, this is a good time to ask (innocently) for one of them to summarize for you what they found. The correct question on your part can cause them to re-examine what they've done without feeling that you've invalidated their answer (point of philosophy: you want them to be able to criticize their own work, realizing that mathematics will determine whether they are right, and that what they discover about this cannot be overruled by the teacher).

Example: A group has quickly concluded that there no solutions, but they have used the incorrect manipulation

$$(a+b)^2 = a^2 + b^2.$$

This might be a good time to hand them a solution and have them figure out how to reslove the contradiction. Don't settle for "Gee, Teacher, your solution works so I guess we did something wrong." Instead, insist that it is you or the laws of algebra at fault. When they are able to pinpoint what went wrong, ask them to prove to you that $(a+b)^2 = a^2 + b^2$ really is not a law of algebra.

If there is only one solution and you don't want to spoil it for them, you may be able to hand them only the part of the solution that contradicts their error. If they are correct that there is no solution but haven't adequately argued it, tell them that there is indeed a solution, namely $a = 51$. When that doesn't work, try 52. Ask them after that whether they plan to do this for all natural numbers or whether they can give you a reason why none will work.

2.2 "Help, we're stuck"

The twin dangers here are that lazy students will say they're stuck so you do the work for them, while students who are truly stuck will lose morale if they have to sit idly during class. Asking if they have any ideas on what they might try will prove embarrassing if the answer is no. Sometimes I do this anyway. Sometimes I replace the problem with a smaller one: if you knew that $A = 15$ could you do the problem? Can you do the problem if you aren't required to make the number of cows and chickens the same? Sometimes I guess why they're stuck: So the problem is you don't really know the definition of average speed? An example of what might happen here is that they did know this but didn't think of going back to definitions as a way to proceed. Now when they say no that's not the problem, we know the definition of average speed, I say, "Oh, then you must be saying you don't have any way of determining the quantities defining speed, such as the time or the distance." They then say how they will proceed on this and I can smile and leave. In other cases, they are stuck because they don't really understand what's being asked. You can ask them to rephrase it, or ask them how they would check if someone else's answer (here you specify it) was right. It helps to have snooped enough so you have a good guess of where they're stuck. If not, you can ask them but won't always get reliable information.

If you have previously addressed problem-solving tactics in this class, then you might suggest to the students that they get out their list of problem-solving tactics and try some. It will do the students more good if you ask them to try a technique than if you tell them more precisely what to do, so try to keep your list of techniques down to a few that almost always help. Doing a problem with numbers instead of variables, doing a problem with smaller numbers instead of the given numbers, or sheer trial and error, are almost always useful.

2.3 Getting groups to work together

The pre-service teachers tend to work well together. The pre-calculus population is less accustomed to this, and groups will degenerate into a lot of individuals ignoring each other, or occasionally explaining to each other. You may at times need to tell them explicitly "Janet has found what she thinks is an answer but Steve and Brenda apparently don't understand what she did, so Janet, you're going to have to explain it and see if you can convince Steve and Brenda." However, you should try other things first, before being this explicit. Ask Brenda what her group has found so far, and don't let anyone else answer for her. If she says she's stuck, ask if her whole group is stuck, and if not, tell her you'll come back in 3 minutes and ask her again for a summary of what her group has done. **Make sure groups are sitting in a tight circle**, not a line or a disarrayed cluster.

Sometimes you can give them specific tasks: Jason, finish the calculation you're doing; meanwhile Ann will add up Rob's numbers and will then check to see if they agree with yours; if not, it's up to all of you to judge which method if either was correct and why. If a group really has bad chemistry, change it. I've had students say to me: I just can't work with Judy - she won't listen and hogs the discussion. In that case, put Judy in with someone smarter or more aggressive than she. When re-forming the groups, I usually randomize again, but if there are trouble students, sometimes I stack the deck so that they get put with students who can handle them.

2.4 Free riders

There are always lazy students who are content to let others do the work. If it's just laziness, I don't hesitate to reprimand them explicitly: "Adam - you're just staring into space and letting the other three figure out the problem; if the problems aren't challenging enough, then I can let you work faster in a group by yourself, but judging from your homework that's not the issue." On the other hand, if it's a student with a confidence problem who needs some nurturing, it's probably better to make a note of it and to continue to ask

that student to explain what their group has done whenever you come around (you have to ask other students sometimes, or it gets too obvious). Involving the student as much as possible, with questions that are at their level but not patronizing, will often cure this. Also impress upon the others that it's their job to make sure the free rider is keeping up with the group since that person (you've just decided) will be in charge of the first group writeup. If a significant portion of the grade is based on exams (individual of course, not group), that will motivate students not to free-ride.

2.5 Staying on task

The less you have to reprimand here, the better. Make sure that when you tell them to get into groups they know exactly what they're supposed to start working on. Make the rounds quickly at the beginning so they don't start chatting, and keep an ear out for it later on. If a group continues to be bad about this, you can watch them, visit them more, chastize them humorously, make sure they don't get grouped together next time, etc., but if the whole class is bad, you should examine what you're doing that promotes doing something other than the math. You could be leaving them in small groups too long, while the slow groups finish. You could be joking with them too much during class time. I do also use reprimands, but sparingly - on Day 3 this year I was in an impatient mood so I reprimanded the two groups that took more than a minute to form their groups and start working. I said "Sounds interesting whatever you're talking about, but you've got to get started on problem 1 - time's short this week!"

2.6 Students who are behind

First, it's a good idea to know what you do for them and what you can't. You can do a lot for these students in office hours, but it's not realistic to be able to spend more than (or even as much as) half an hour a week outside of class with any one student. So if they need help on a regular basis, suggest that they arrange for tutoring. You should find out

before the semester wht such resources are available. If they're doing passing work, but still underconfident, point out to them that if they continue to work at this level, they'll pass the class. Don't however make promises you can't keep about their grades; it's best not to prognosticate before they've taken an exam.

All that being said, you have to make sure they get the most out of their group work and don't drag down the group (they are as much afraid of this as you are). Here are some ways to build their confidence and make them more likely to participate to advantage. Spend some time around their group and be ready to pounce on those times when the lagging student, "Lenny", comes up with a good idea. Assign credit: if Jocelyn figures out how to do something with Lenny's idea, then it's "Lenny & Jocelyn's method". Try to give them constructive comments on their homework (I try to do this for everyone, but when time does not permit this, concentrate on students who need it). Making sure they are keeping up with their group is delicate - asking them to explain to you where there group is will help if they're not too far lost. Once they are far behind, consider placing them next time with a group of students that talk a lot, don't go all that fast, and are as kind as possible. At worst, you may have to settle for Lenny participating minimally during class and trying to catch up on his own at home.

Select Lenny to give a presentation, either solo or on behalf of his group. On a one-time basis you can invest a little extra time to help him with this to make sure it goes over well (have him practice it on you till he's confident enough to field questions).

Avoid asking Lenny really easy questions. This will make him feel like he must be dumb. These questions are scary in general since the reward for a correct answer is almost zero and the penalty for an incorrect answer is large. At your discretion, you might chose Lenny to answer questions that are not black and white, asked of the class at large: which of these problems was the hardest?, what kinds of thinking did you think were necessary on this problem?, and so on. This does not in itself teach Lenny the material, but it keeps him from feeling like a ghost and reminds him that his opinion matters.

2.7 Students who are ahead

Having such a student can be a real boon if they are gifted teachers as well. If they have a good feel for how to explain things and help others, they will make your class run more smoothly than you can on their own. Even in this case, **avoid treating them in front of the class as a reliable source for right answers**. You don't want to create a situation where calling on them is tantamount to telling the class something yourself. It is OK though, to treat them as a reliable source for intelligent commentary.

If a student is obviously head and shoulders above the rest from Day 1, you may consider exempting them from the course. That is, if they can do the worksheets on their own then they can probably pass the exemption exam, go on to the next course, and leave you with a more homogeneous class. Recommend that they see the course coordinator for an exemption exam; everyone will benefit from this. Later in the semester this is less of a good idea, though I've done it.

Assuming "Einstein" stays in the class and is not self-policing, you need to keep an eye on Einstein's group to make sure that Einstein is not explaining things to the others before they have a chance to figure it out themselves. Let Einstein explain things at the board in situations where you know there will be some wrong or unclear stuff in the explanation. Make sure though, that you give Einstein as much encouragement for what was right and clear as you would another student. If Einstein is a loner and tends to work fast but not share with the others, that will probably work out fine. You can explicitly designate Einstein as a group of 1 next time, or simply allow a *de facto* group of 1 to form. Sometimes, you can try asking Einstein explicitly to figure out a hint to give the rest of the group as to how to proceed but that won't completely solve the problem for them. It will make Einstein summon up teaching skills that are worthwhile in general, so it's worth a try, but be aware that Einstein may not be capable of this. In any case, don't let students disparage themselves in comparison to Einstein. You can say, "I see, because Einstein solved this problem in 5 minutes and you can't, you're going to give up?"

3 Managing Socratic discussions

This is the hardest part, and the part of teaching SC classes that improves the most with practice. When observing someone else's Socratic discussions, try to imagine what would have happened had they made different choices (told or not told the students something, came up with a good counter-question, decided to pursue or not to pursue a student's line of reasoning).

3.1 Dead ends

When an idea is proposed, the instructor will usually know right away whether it will lead anywhere. If it won't, there is a strong temptation to discourage the students from pursuing the idea. This may be a mistake. Probably the best thing that can happen in a Socratic discussion is a *flaming dead end*, meaning that an incorrect line of reasoning leads to a consequence so patently false that the students are compelled to re-examine the road that got them there. If you see your students headed for one of these, then all you really need to do is encourage them to get there without undue delay. Some things you might want to do are: get them to explicitly reaffirm the wrong assumption, so that they will remember it later and be able to pinpoint it; shut down any further sidetracks that branch off of this one (e.g., "OK, that's a good idea, but first we'll finish pursuing this one"); hasten their demise by keeping the pace brisk, perhaps doing some of the arithmetic for them or providing clarifying paraphrases.

You can influence how flaming a dead end will be by making the issue more concrete: ask them to illustrate their result with actual numbers (e.g., "so if the initial weight was 250 grams, then we see the final weight of $400 - 2w$ comes out to be what? Oh, I see, -100 grams; rather on the light side...").

Perhaps you will need to summarize their findings, juxtaposing two findings that are contradictory, or in the case that they have contradicted some of the given information, you

may need to restate the givens by saying e.g., "so you have now proved that the only whole number between 100 and 500 having no two digits the same and satisfying blah blah blah is 337." To further rub it in, it often works well to insist that you believed their method and there must be some other mistake: "Ah, you've proved that the long division algorithm is wrong", or "Ah, you've proved that when you use variables with subscripts, the usual laws of algebra don't apply" (if you can trust them to fight back on this one).

Other kinds of dead end are less useful. Perhaps they are following a reasonable line of inquiry but it doesn't get them anywhere: looking for a nonexistent pattern, introducing too many variables, classifying according to an ineffective scheme. A reasonable goal in this case is to get them to figure out that they're stuck. If you tell them (or indicate in any of 100 nonverbal ways) that their idea won't work, they will learn to look to you for validation of their ideas, whereas if they reach a dead end themselves and consciously decide to look for another approach, they have learned something valuable about problem-solving. That being said, there are ways to reduce the amount of time spent following a dead end. One trick is to decide after hearing a suggested approach whether to follow it immediately or whether to treat it as one of many to be written on the board before the class decides which to follow. If Jenny reports finding a pattern starting $4, 6, 8, 12$ and reports her reasoning as to what is likely to come next, you get to choose between (a) getting the whole class involved in speculating about the next number or (b) writing on the board "Idea: look for a pattern". The key feature of this trick is that you're not giving anything away. Approach (a) is reasonable in some contexts, where the discussion of pattern promises to have some depth, and more importantly, approach (b) is something you sometimes use when the approach offered is correct. In fact you should make sure to use (b) on occasions where there were multiple interesting approaches but the first one offered happens to be the best: you catalogue every group's approach before asking the class to pick one and follow it.

Another way to expedite matters is to insist that the goals be well defined. Often when a bad approach is put into words, it comes out sounding discouraging: we thought we'd name as many variables as possible and then hope that inspiration struck; we decided that if n was equal to 5 the solution was obvious but we don't know how to do it for any other

value. Sometimes mild discouragement doesn't work. I remember a worksheet designed to get them to invent the binary number system by asking them to come up with a scheme for representing all numbers with 1's and 0's. This cost a full day of discussion of the relative merits of various schemes, none of which had anything to do with binary. The instructor was very successful that semester, and in my opinion the investment of that day paid off when students continued to work hard throughout the course because they didn't feel that the instructor was going to provide the answer for them. This takes guts, and doesn't work too well if the instructor conveys a growing uneasiness about the whole project. So if you're not up to following the wind, you're probably better off cutting your losses, as in the next paragraph.

The least promising dead end is a total lack of ideas. Probably it's best not to convene a discussion at this point but to continue working in small groups where you can ask questions that elicit further work and break the impasse. But suppose a class discussion on a certain problem fizzles out midway. This might be a good time to drop it. If it's not essential that they end up knowing how to do the problem, and they don't have a realistic shot at finishing it for homework, perhaps make it into an extra credit assignment. If it is essential, consider dropping it for now and writing a worksheet for the next class that will lead them to it in more manageable steps. You probably need more time to solve this problem than you have on the spot.

3.2 How to listen

You need to listen to students and they need to listen to each other. Tom Lester once told me of a study showing that the average amount of time between when a teacher asks a student a question and when the teacher prompts the student or gives up on them is 2 seconds. Two seconds is longer than it sounds, but nowhere near long enough to formulate a coherent thought unless you were already thinking it before the quesiton was asked. There may not be anything you can do about the sound-bite trend in TV reporting, but there's a lot you can do about it in your classroom. The first thing to try is waiting. Don't nod

yes or no, or say uh huh, or give the student any feedback at all until they have finished saying what they wanted to say. Then wait five or ten more seconds. The odds are that the student will, after pausing for breath, realize that they are not finished and continue. If not, at least the other students will have had a chance to think about what they just heard. If you're uncomfortable with this long a pause, try pacing or holding eye contact with the respondent as if you expect them to continue, or act as if you're trying to digest what they've just told you. In fact often you really will need time to think. If they said something that was wrong in a puzzling way, see if you can figure out what they really meant. Students will only listen to each other if you set an example, so make sure you don't respond without having really heard.

Students are also more likely to listen to each other if they feel that they are responsible for having understood it. In small groups they are more likely to feel this, but at a ratio of twenty or thirty to one, many feel that they can just take notes and sort it out later or not at all. They may also feel they have no right to interrupt since everyone else obviously understands. You can counter this by demonstrating an expectation that each student understands what each other student has said. After one student says something the slightest bit unclear, ask another to repeat it in her own words. This is a good time to pick on students rather than have them raise their hand to volunteer a paraphrase. If student B can't paraphrase what student A said, it's not necessarily student B's fault. Student B can ask student A to clarify if necessary, or ask for volunteers for someone else to clarify. Make sure to go back and find out whether student C's clarification of student A's remark did in fact help student B. After a little experience you'll know better when to go through this routine. If student B simply wasn't listening, they might feel reprimanded, but that's OK. It doesn't really work when the remark was clear in the first place, although it doesn't hurt to get a quick affirmation from the whole class that it is clear so far. The basic standard you are setting is that the discussion involves the whole class and is not a collection of one on one dialogues between the teacher and individual students.

3.3 Staging

Your expectations of the nature of a class discussion are conveyed nonverbally as much or more than verbally. Most instructors ask the students to rearrange their desks into a large U shape for any but the briefest class discussions. A subtler but important technique is to put as much of the class as possible between you and the respondent. If you call on a student on the left side of the room, walk over to the right side as you're doing so. As the words flow between you and the respondent, the almost physical presence of a stream flowing between the two of you will wash over the students in between. You and the respondent also keep eye contact with the rest of the class this way.

It is often a good idea to get students to come up to the blackboard. Students will give longer monologues at the blackboard, so be prepared to be a more active moderator if the student is losing the rest of the class. Be careful not to make having the right solution a pre-requisite for coming to the board, lest the students stop thinking critically and accept any blackboard demonstration as a surrogate for your telling them something. When a student is at the board, I try to take up a position in the back or on the perimeter of the room. Sometimes I sit in the student's seat. This has the effect of including the rest of the class, as above, and also gives me a new vantage - you'd be surprised at what you see this way.

Other body language to be aware of is whether you are passing judgment on what you hear. Do your eyes flit impatiently with wrong answers? Do you gesture in agreement with right answers? Do you angle your body to the board as if to write down something correct, then pause if it's not what you wanted? If you've chosen to teach SC style classes, it's because you want the students to develop their own judgment, so avoid this kind of tip-off.

A related topic is the use of intentional errors. These are a hit with kids, in a slapstick sort of way, but adult students tend to feel patronized. Instead, I substitute the mischeivous lie. If a student tells me the found all five regular polyhedra I may say, "Ah, so you haven't found the other two then?" They can often sense that I'm putting them on, but will still

take the bait and try to prove that there aren't any more. On a problem best solved by assigning a variable to a certain quantity, I once told a group of frustrated students that I'd tell them the value of one quantity for free if they could decide which quantity they wanted to know. I planned to lie and tell them it was 10 when it was really 6. In fact I had tried this previously with success: the students figured out that the assumption of 10 led to a contradiction and were able to figure out the unique value that didn't lead to a contradition. This time it was even more successful. By the time I came back to give them their free question, they had figured out what quantity they wanted to be told, had put in x for this, and had gotten the solution (well ahead of the rest of the class).

3.4 Asking the right questions

When my teaching is evaluated by my students, they often say that I never answer questions, or answer them with another question. I take it as a complement even though it isn't meant as one. The most common such interchanges are

> Student: Is this right?
> Instructor: I don't know. Does it sound right to you? Can you elaborate?
>
> Student: What should we do from here?
> Instructor: What do you think? Does anyone have any ideas?
>
> Student: Can we say blah blah blah?
> Instructor: I don't know, can you?

There is a certain amount of this you can get away with, depending on your personality and theirs. If you start sounding like "Eliza"[1], you won't get good results. Instead, try to ask them useful questions related to the specifics of what they've said. In the three above scenarios, try respectively

[1]The computer program imitating a nondirctional therapist, an early (and crude) approximation to something that could past the Turing test.

Are you asking if your computation is correct, or if it will prove useful?

Is there a problem-soving strategy that you know that might work here? Why don't these equations tell you what x is?

If you're wondering whether you can assign the variable z to be the average of all the prices, the answer is yes, but you haven't yet said whether we know anything about z.

When observing other classes, this is where you should let your imagination run free. Imagine what questions they might have asked. You hindsight now will be your foresight tomorrow.

The question "do you understand?" is the most often abused. (Notice that this virtually the only question in the repertoire of the conventional lecturer and rarely elicits an honest response.) Some better variants are: can you say that in your own words? could you do what John just said with different numbers? do you agree or disagree? in what way is this similar to what so and so did? These are all listening-comprehension questions, which will tell you whether a students has taken in the meaning of what was said more reliably than if you ask the student to report a Yes/No answer.

Good questioning can help to reach flaming dead ends. Ask what happens when $x = 5$, or whether their purported method works for all starting data and not just what was given. If a student gives a vague definition, find a borderline case and ask how their definition applies in that case. Try also questions that goad by disingenuity. If their method is more general than they realize, pretend they got lucky (ham it up!). Ask how they got lucky enough to try their method on a square rather than a pentagon or hexagon for which it "probably wouldn't have worked"; summarize for them that "because n was a multiple of 10, you were able to find k and l to fit the information given."

A final point to emphasize is: **don't make your questions too easy**. Often instructors who are having trouble getting students to speak up will tone down their questions, and

are then puzzled that the chilly atmosphere doesn't improve. A timid student sees a too-easy question as a no-win situation. Either the student gets it right, which is not really impressive, or gets it wrong which is doubly embarrassing. It's better to make the reward higher. Long-term confidence is built by the realization that you can answer something nontrivial. And don't forget there may be students who are being quiet because they're bored; they will certainly appreciate your finding an alternative to going slower.

There are ways to construct nonthreatening questions that are suitable for tempting the timid out of their shell. Ask them to guess something that they don't have much of a basis for guessing yet, and get half a dozen different answers on the board to emphasize that it's still guesswork at this point. Ask others to agree or disagree with any of the guesses and discuss their relative merits. Ask them to help you read each other's mind: "Ah, 16 is an intelligent guess - can you see why Jackie thinks the answer might be 16?"

3.5 Order versus chaos

Ideally your students will be eager to answer your questions and discuss their ideas, but will listen patiently and attentively to each other and to you. If students are not willing to speak up and discuss their ideas, you need to loosen them up. It is a bad sign, for example, if the students are not happily chatting away when you enter the room five minutes before class, and are sitting in silence or whispering. In this case, you have probably done too well at eliminating chaos. Try assigning an activity in small groups where different groups are doing different things and they need to walk across the room to share information with each other. For example, there's a worksheet in Math 112 on infinite series where they approximate numerically some infinite sums and try to form conjectures based on each other's conclusions. Assigning a group project where they have to work outside of class together can tighten the bonds and make people feel more comfortable talking. When leading class discussions, be freer and more willing to follow the students ideas wherever they lead. Dispensing with handraising and having students just call out can quicken the pace. So can calling on students law-school style. If you can provide a supportive atmosphere for

their answers - not being phony about what a nice try their wrong answer is, but being able to isolate and bring out the grains of truth in what they said - then this can be just the medicine for shy students.

A classroom that's too chaotic is a problem also. If you have to call the students to attention more than once before they pipe down and listen, or if there is crosstalk during class discussions, you probably ought to do something about it. You can address this explicitly, asking the students to pay attention to you and to each other. You have to be consistent about this or they won't believe you mean it. Indirect methods of dealing with this are, however, usually more effective and should be tried first or at least in parallel. Insist on an orderly formation of desks into a U shape before a class discussion, rather than having them minimally pertutrb the small-group seating arrangements. It takes an extra minute, but it's worth it. In fact tell them you're pressed for time so they have to rearrange the desks in 30 seconds. A snappy set change will set the tone for what follows. When you observe crosstalk, try to get one of the crosstalkers up to the board to explain something, or to comment on what's just been said. By maximally involving that student in the lesson, you'll eliminate most of the off-task crosstalk, and the on-task crosstalk can probably be lived with. Another chaos reduction technique is to give them a more rigid idea of the structure of each class. Say you're going to spend 16 minutes in groups before convening a class discussion and then stick to it with absurd precision. The more aware they are of the structure of the lesson, the more they will stick to the tasks at hand.

The main point of this section is that you should make a conscious effort to optimize your position on the order-chaos axis, and that increasing order or chaos in the physical arrangements or chronological structure or types of assigned activities can help you change the balance in your class discussions.

4 Curriculum

4.1 A vision

Those of us who have invested many hours inventing and plagiarizing the worksheets and problem sets for SC courses have an advantage when it comes to day-to-day curricular decisions. We know where we're trying to get, and if something isn't working, we know whether we need to push it, to try another tack, or whether we should just drop it. Once the set of course materials becomes more standardized, instructors inevitably lose flexibility. Some of the flexibility is lost for good, insofar as any of the subject matter fulfills prerequisites for other courses. But you can keep a lot of the flexibility if you thoroughly acquaint yourself with each worksheet before the semester begins. That way you'll know exactly what follows on what, you'll have an idea of what's hard and what's easy, and where the light is at the end of a tunnel through particularly difficult material. The only way I know to do a sufficient job of this is to personalize the coursepack. Choose a sequence of sections and problem worksheets that makes sense to you. Look for what might be a poorly conceived worksheet and write a replacement, or fill in gaps you might notice.

In tandem with personalizing the coursepack, you should develop (if you haven't already) your own vision for the course. What are you trying to accomplish and what is the role of each section of the curriculum? If you don't have an idea of this, you will continually encounter awkward moments in class where you're trying to get students to understand something and start feeling like you're just making them jump through hoops. If you have firmly in mind not only the underlying concept but what about it they really need to know, you'll be able to devise a new approach on the fly (or more honestly, if you don't then you certainly won't). In the 130-131-132 sequence, students tend to broach openly the question of why they're being made to learn such and such, which you will also have trouble with if you don't have your own vision for the course. Hopefully we've created enough raw material so that personalizing the coursepack won't take anywhere near the time it took during the pilot versions. In pre-calculus, the students are enrolled for a variety of different reasons,

and it will probably help you to survey them on the first day to find out why they're enrolled.

4.2 Pacing

Another advantage we had during the pilot versions is that we were defining the expectations as to what could be covered in a semester, and so automatically lived up to them. Now that there are norms for the course, there is pressure to keep up with them. If you find you're going "too slowly", it's worth figuring out why, but you should resist the temptation to hurry. If you start lecturing, forcing ideas on the students, letting the fast ones serve as surrogate expositors, and so forth, their rate of learning will go down, not up. Instead, consider the following alternatives. (1) Perhaps you've got a slow class. In this case you should probably go at the pace they need and cover less. They'll be better off in the long run. But if this happens to you again the next time, try a different tack! (2) You may be underassigning homework. If you ask them more often to finish trying to understand a problem sheet on their own at home and hand it in for a writeup, they will do it. You'll have to be careful to stay in touch with their needs, but if you assign the homework for a date that allows them to come back and ask for more class time or office hour time if needed, they should be able to handle the responsibility. (3) You may be covering the material more thoroughly than intended. When I write a worksheet, I often include extra, more difficult problems at the end that I plan to use only if time permits (which it seldom does). Use your judgment as to what portion of a worksheet is "par for the course" and compare with other instructors. (4) Your efforts to be Socratic might be too extreme. Deciding whether you're telling the students too much or too little can be like figuring out whether a plant is over- or under-watered. You'd best check with an expert.

As in proper questioning, a good worksheet needs to be nontrivial. If they're going through entire worksheets ahead of schedule, then they're probably not being challenged enough (although this is a great morale booster when it happens occasionally). If you're constructing a significant amount of new material for your course, recognize the warning signs of too easy material, since they're subtler than warnings about the opposite. A

general loss of interest, low variance in homework scores, a lack of good places to begin class discussions, all indicate that either your materials are not meaty enough or that you're doing too much of the students' work for them in class.

4.3 Resources

For the 130-131-132 sequence and for 112, we hope to have in your hands a set of worksheets containing as subsets several different possible choices of curriculum. We also will eventually supply a set of instructors' notes, commenting on how each worksheet has worked (or failed) in the past, what the students got out of it, how the discussion went or should have gone, what advance preparations are advisable, and so on. We'll try to compile a file of problems for use on exams, as well as the actual past exams. Various hands-on materials are owned by individuals in the department and we'll try to supply a detailed list of those as well. Meanwhile, take note that Bleicher, Crowe and Pemantle have some fun toys, that the Instructional Materials Center has a lot of good stuff, and that if there's something really cool you want to buy, the department chair might be able to come up with some money for you.

5 Organization

SC style classes have more inherent disorganization than traditional classes, which is why you need to pay particular attention to organization.

5.1 Your records

Students, at Wisconsin more than at other places I've been, react extremely positively to the appearance of organization. Probably the pre-service teachers are particularly impressed

by this since they are consciously judging you on your pedagogical techniques. **Be a compulsive record-keeper.** I find it very helpful to reserve the 20 minutes after each class for writing a short summary of what happened in class that day. That way, if I've told a student I'd find them an extra-credit assignment, or if I've promised to bring something to class next time or promised that the next class woul begin with a discussion of something, I can write this down along with other notes as to what I have in mind for the next class. In an SC style class there is a greater opportunity for unexpected things to happen, therefore a greater need to write down what did happen.

Keeping date records of the homework you've assigned, both due dates and the date it was assigned, is essential. Whether or not you accept late homework is up to you, but it is certainly better to have students ask solicitously in advance for you to accept their late papers, which you will probably grant, than for students to assume it's OK and be upset if you don't grant them an extension after the fact. They are more likely to do the latter if they get the idea that you yourself don't remember when the homework was due. In fact, if I arrive at class early, I often take the opportunity to write up a reminder of what is due when.

5.2 Grading

The message here is the same as in the previous section. Include in your course packet a clearly defined grading policy, specifying what portions of the grade are from homework, exams, quizzes, projects, group work, attendance, or whatever else you grade on. Give it enough thought so that you remember it easily, and can answer their questions immediately. If historical grade distribution data is available, you would be well advised to stick to it, since this will help to allay fears that the unusual pedagogical style will adversely affect their grade.

If you find yourself spending too much time grading the students' assignments, then you're in good company. They learn a lot from this, so I encourage you to do as much as

you can stomach in the way of grading writeups. That being said, there are ways to reduce the workload a little. Every now and then you can assign a group assignment instead of an individual assignment. Don't do this more than once between each change of group assignments, and make sure the assignment is appropriate for a group writeup. Realize that it is hard for them to find time to meet outside of class, so ideally the assignment should combine what they were able to work out together in class with some labor they can easily divide and do at home. Another useful tactic is multiple choice questions on exams. I always include multiple choice or True/False sections on exams, since the purpose of these is evaluation and not instruction. Of course you need to save room on the exams for questions where more thorough explanations are demanded. Another useful device is to assign only 1/3 to 1/2 of the worksheets for writing up. Don't tell them which will be written up until you've spent all the class time you're going to spend on them, then assign the worksheet as a writeup or don't.

5.3 The bell

When the bell rings at the end of class, everything becomes exponentially harder. My advice is to watch the clock like a hawk, so that you can make sure to wrap up the discussion at 2 minutes before the end. The discussion usually leaks over an extra minute, giving you one minute to say any summary comments or give instructions on homework, etc. This can be an important routine even when you have little to say: it makes you seem organized and on top of things.

If you can tell you are going to want to go overtime, because of a red-hot discussion you want to complete or something that's necessary so they can do their homework, announce to them 5 or 10 minutes ahead of time that you will probably be going overtime. It's best if you've let them go a minute or two early once before and have mentioned at the time that you're banking those minutes for such an occasion as this. If they're working in small groups at the end of class, it's less crucial but you still may want to halt them 1 or 2 minutes before the bell for closing remarks - it's even harder to get their attention after the bell

when you don't already have the stage.

In short: **don't ever be surprised by the bell**.