0.1 Dead ends

When an idea is proposed, the instructor will usually know right away whether it will lead anywhere. If it won't, there is a strong temptation to discourage the students from pursuing the idea. This may be a mistake. Probably the best thing that can happen in a Socratic discussion is a *flaming dead end*, meaning that an incorrect line of reasoning leads to a consequence so patently false that the students are compelled to re-examine the road that got them there. If you see your students headed for one of these, then all you really need to do is encourage them to get there without undue delay. Some things you might want to do are: get them to explicitly reaffirm the wrong assumption, so that they will remember it later and be able to pinpoint it; shut down any further sidetracks that branch off of this one (e.g., "OK, that's a good idea, but first we'll finish pursuing this one"); hasten their demise by keeping the pace brisk, perhaps doing some of the arithmetic for them or providing clarifying paraphrases.

You can influence how flaming a dead end will be by making the issue more concrete: ask them to illustrate their result with actual numbers (e.g., "so if the initial weight was 250 grams, then we see the final weight of $400 - 2w$ comes out to be what? Oh, I see, -100 grams; rather on the light side...").

Perhaps you will need to summarize their findings, juxtaposing two findings that are contradictory, or in the case that they have contradicted some of the given information, you may need to restate the givens by saying e.g., "so you have now proved that the only whole number between 100 and 500 having no two digits the same and satisfying blah blah blah is 337." To further rub it in, it often works well to insist that you believed their method and there must be some other mistake: "Ah, you've proved that the long division algorithm is wrong", or "Ah, you've proved that when you use variables with subscripts, the usual laws of algebra don't apply" (if you can trust them to fight back on this one).

Other kinds of dead end are less useful. Perhaps they are following a reasonable line of inquiry but it doesn't get them anywhere: looking for a nonexistent pattern, introducing

too many variables, classifying according to an ineffective scheme. A reasonable goal in this case is to get them to figure out that they're stuck. If you tell them (or indicate in any of 100 nonverbal ways) that their idea won't work, they will learn to look to you for validation of their ideas, whereas if they reach a dead end themselves and consciously decide to look for another approach, they have learned something valuable about problem-solving. That being said, there are ways to reduce the amount of time spent following a dead end. One trick is to decide after hearing a suggested approach whether to follow it immediately or whether to treat it as one of many to be written on the board before the class decides which to follow. If Jenny reports finding a pattern starting $4, 6, 8, 12$ and reports her reasoning as to what is likely to come next, you get to choose between (a) getting the whole class involved in speculating about the next number or (b) writing on the board "Idea: look for a pattern". The key feature of this trick is that you're not giving anything away. Approach (a) is reasonable in some contexts, where the discussion of pattern promises to have some depth, and more importantly, approach (b) is something you sometimes use when the approach offered is correct. In fact you should make sure to use (b) on occasions where there were multiple interesting approaches but the first one offered happens to be the best: you catalogue every group's approach before asking the class to pick one and follow it.

Another way to expedite matters is to insist that the goals be well defined. Often when a bad approach is put into words, it comes out sounding discouraging: we thought we'd name as many variables as possible and then hope that inspiration struck; we decided that if n was equal to 5 the solution was obvious but we don't know how to do it for any other value. Sometimes mild discouragement doesn't work. I remember a worksheet designed to get them to invent the binary number system by asking them to come up with a scheme for representing all numbers with 1's and 0's. This cost a full day of discussion of the relative merits of various schemes, none of which had anything to do with binary. The instructor was very successful that semester, and in my opinion the investment of that day paid off when students continued to work hard throughout the course because they didn't feel that the instructor was going to provide the answer for them. This takes guts, and doesn't work too well if the instructor conveys a growing uneasiness about the whole project. So if you're

not up to following the wind, you're probably better off cutting your losses, as in the next paragraph.

The least promising dead end is a total lack of ideas. Probably it's best not to convene a discussion at this point but to continue working in small groups where you can ask questions that elicit further work and break the impasse. But suppose a class discussion on a certain problem fizzles out midway. This might be a good time to drop it. If it's not essential that they end up knowing how to do the problem, and they don't have a realistic shot at finishing it for homework, perhaps make it into an extra credit assignment. If it is essential, consider dropping it for now and writing a worksheet for the next class that will lead them to it in more manageable steps. You probably need more time to solve this problem than you have on the spot.