0.1 The philosophy

Unless you've been on Mars, you're probably aware of the age-old battle between "skills" advocates and "process" advocates. I'm from the "process" camp but I hope to avoid a lot of the partisanship that is prevalent in discussions of pedagogy and stick mainly to points both sides agree on. We want students to come away from (lower level) math classes with certain skills and attitudes. In particular we want them to reason and prove, to translate between words and symbols, to perform algebraic manipulations correctly and with understanding of the justification, and to solve problems other than clones of problems they have been shown how to do. Whether or not you believe specific skills and knowledge to be paramount (arithmetic of negative numbers, solution of quadratic equations, propositional logic, summation of common series), you undoubtedly want them to know these things in the ways mentioned above: verbally and symbolically, with justification and proof, well enough to apply to new situations.

The tenet underlying SC classes is that students need to learn that they can think for themselves, and that they will be able to learn properly if and only if they are forced or enticed to continue thinking things through on their own. Realize please that this does not apply to students who already have this skill. I don't think we need SC classes at the advanced undergraduate level, and they become increasingly inefficient at higher levels. If college admissions standards (or high school graduation standards) were what we'd like them to be, we wouldn't need SC classes in college at all. The students who benefit from SC classes are ones I would term remedial: those taking pre-calculus, and those in the Ed program who are required to take what is essentially junior high school mathematics.

Our philosophy with these students is to do anything we can to get them to think and speak mathematics, and then once we have them going, to exact from them a quality of mathematical reasoning that is higher than anything ever asked of them in a traditional course, thus ensuring that they learn the course content in a useful and permanent way. The meta-skill we emphasize is for the students to **know when they know something**, versus when they are just guessing or are confused. In the time-span of the course or sequence of

courses we move from a "process is everything, choice of content matters little" approach to a stage where we cover the traditional content and expect students to focus on these topics and skills while applying the critical thinking they have learned in the first phase. The first phase is the harder phase for most instructors, since we have to be psychologists - and sometimes mind-readers - in addition to being mathematicians. These notes concentrate on this phase, though they apply to the other as well.

Perhaps the most controversial part of this approach is our unwillingness to tell students the answer. Some skills can only be acquired by imitation, so it is possible to be overly dogmatic on this point. The basis for our approach is that much of this material is accessible to these students, with a little help from us, and that our habitually providing answers will cause their problem-solving ability to atrophy, though it may increase their rate of skill acquisition (I would argue that it won't). Thus we make every attempt when discussing an attempted solution in class not to tip off whether it is right or wrong until the whole class has had a chance to criticize it or register comprehension and agreement. Depending on the context, we do or do not in the end provide model solutions.