

Math 5810, Spring 2025, problems on asymptotics

1. Strengthen the well known approximation $\sqrt{1+\epsilon} \approx 1 + \epsilon/2$ when ϵ is small to an asymptotic statement and say how this follows from Taylor's theorem.
2. Find an asymptotic expression for $\log(n+j) - \log n$ as $n \rightarrow \infty$.
3. Find an asymptotic expression for $\sum_{j=1}^k \log\left(1 + \frac{j}{n}\right)$. You can assume k is small compared to n , provided you make an explicit assumption about how small you want to assume k is with respect to n .
4. What can you say about the asymptotics of $\int_0^\infty x^b e^{-\lambda x} dx$ as $\lambda \rightarrow \infty$, where b is a parameter? You are welcome to make assumptions about b .