Math 110, Spring 2015
HWK10 due WED 15 April

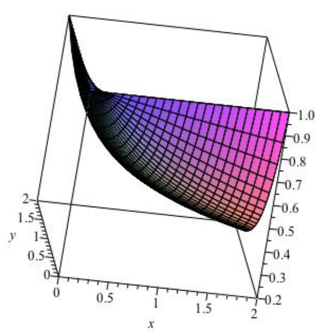
1. Three functions are given, then three graphs then two contour plots.

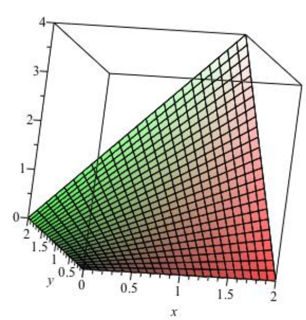
   (a) Match each function to a graph and a contour plot.

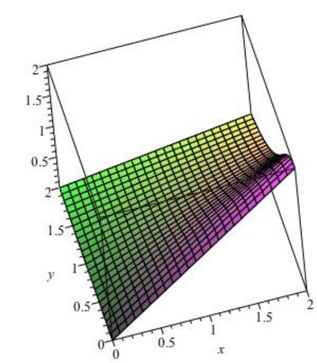
   (b) Explain why two of the functions have the nearly identical contour plots.

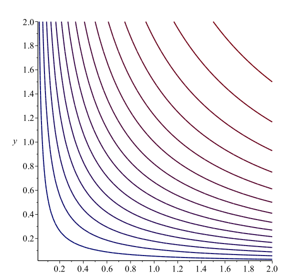
   (c) On each contour plot, indicate with the name of the function and an arrow roughly the directions in which the function is increasing.

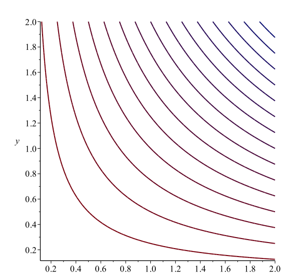
$$\begin{aligned} f(x,y) &= \frac{x}{1+y^2} \\ g(x,y) &= xy \\ h(x,y) &= 1/(1+xy) \end{aligned}$$

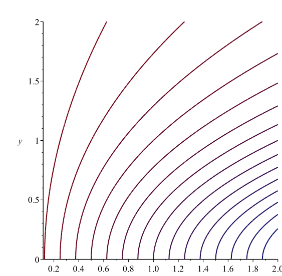












2. In each case, do these five things:

   (i) Draw the region R.

   (ii) Write the region as $\{(x, y) : \cdots\}$ in a description corresponding to vertical strips.

   (iii) Write $\int_R f(x, y)dA$ as a double integral of f with limits of integration corresponding to vertical strips and the dx and dy in the right order.

   (iv) Write the region as $\{(x, y) : \cdots\}$ in a description corresponding to horizontal strips.

   (v) Write $\int_R f(x, y)dA$ as a double integral of f with limits of integration corresponding to horizontal strips and the dx and dy in the right order.

   (a) The region under the parabola $y = 5 - x^2$ but above the x-axis.

   (b) The region inside the unit circle in which the value of $x + y$ is positive.

3. Compute these iterated integrals.

(a) $\displaystyle\int_0^3 \int_{-2}^5 1 + x + x^2 y + y^3 \, dx \, dy$

(b) $\displaystyle\int_0^1 \int_0^{1/(1+y^2)} y \, dx \, dy$

(c) $\displaystyle\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 1 \, dy \, dx$

[Hint: draw the picture, then try to avoid any computation.]

4. Compute the double integral

$$\int_0^1 \int_x^1 \frac{2x}{1+y^3}\, dy\, dx$$

by using Fubini's theorem to write it as in integral in the other order. You will need to draw the region R of integration in order to make sure that you write correct limits of integration when you switch the inner and outer variables.

5. A mining site in the shape of a square 100 meters on each side has a river along its southern border. Regulations limit excavation at any point to a maximum depth equal to the distance to the nearest river. Due to a million year old meteorite impact at the southwest corner of the mine, there is some gadolinium in the soil whose concentration varies inversely with the distance to the southwest corner. The amount of extractable gadolinium per unit area is proportional to the concentration and the maximum drilling depth.

(a) Write a formula for the amount of extractable gadolinium per unit area G in terms of the location of the point. The location should be parametrized by a pair of real numbers x and y; you need to state the meaning of x and y and give a formula for $G(x, y)$. Any constants must be named and their units given. A picture would probably help.

(b) Let T be the total amount of gadolinium extractable from this site. Write a double integral expressing the value of T.

(c) When this double integral is computed will it be a number or will any variables or constants remain in the expression?

(d) Give an approximate value for T by dividing the site into a grid with 2 squares in each direction and using the midpoint of each square as the point at which to evaluate G for the double Riemann sum.

(e) Compute the double integral exactly by doing an iterated integral. [Hints: (i) The inner integral is easier if you pick the right variable to go first. (ii) The outer integral is in not in the table on Page 457 but it is in the brief table of integrals at the back of the book.]

6. A pair of random variables (X, Y) has probability density $Ce^{-x/3}$ on the semi-infinite strip $0 \le y \le 2, 0 \le x < \infty$.

   (a) Draw the semi-infinite strip.

   (b) Compute the normalizing constant, C.

   (c) Compute the mean of the X-variable.

(d) Compute the mean of the Y-variable.

(e) Draw the region in the strip of points such that $x + y \leq 2$.

(f) Compute the probability that $X + Y \leq 2$.