Math 110, Spring 2015
HWK09 due FRI 03 April

1. (Problem # 38 in Section 7.2): The half-life of Polonium-210 is 139 days. You obtain a small amount of it, and need to use it before it is 95% gone, after which there will no longer be enough to be useful for its intended purpose. For about how many days will you be able to use the polonium?

Bonus question: unscramble the letters to see for what purpose the polonium was to be used: OOPSNI A NARSUSI TANGE.

2. (Problem # 36 in Section 7.2): To encourage buyers to place 100-unit orders, your firm's sales department applies a continuous discount that makes the unit price a function $p(x)$ of the number x of units ordered. The discount decreases the price at the rate of 1% per unit ordered. The price per unit for a 100-unit order is $p(100) = \$20.09$.

   (a) Find $p(x)$ by solving the following initial value problem:

   $$\frac{dp}{dx} = -\frac{1}{100}\,p\ ;\ p(100) = 20.09\,.$$

   (b) Find the unit price $p(10)$ for a 10 unit order and the unit price $p(90)$ for a 90 unit order.

   (c) The sales department has asked you to find out if it is discounting so much that the firm's revenue $r(x) = x \cdot p(x)$ will actually be less for a 100 unit order than, say, for a 90 unit order. Reassure them by showing that the maximum value of $r(x)$ occurs at $x = 100$.

   (d) Graph the revenue function $r(x)$ for $0 \leq x \leq 200$.

3. Solve the differential equation you wrote for Problem #8 on the differential equations word problem worksheet:

> Money is deposited in a bank account with an annual interest rate of 3% compounded continuously. What is the amount in the account at time t after an initial deposit of \$1000, if money is being added to the account continuously at a rate of \$500 per year, and no withdrawals are made?

4. Solve the differential equation for one of the mango juice problems in the differential equations word problem worksheet. Problem #6 is easier, but if you choose to do #7 instead and get it right you will get 50% extra credit on the problem.

5. When the interest rate is reasonably small, the relation between the continuous rate r and the annualized rate A can be approximated by a Taylor polynomial. Compute the linear and quadratic Taylor polynomials for each in terms of the other.

   (a) The linear approximation for A as a function of r when r is near zero is given by

   $A(r) \approx P_1(r) =$

   (b) The quadratic approximation for A as a function of r when r is near zero is given by

   $A(r) \approx P_2(r) =$

   (c) The linear approximation for r as a function of A when A is near zero is given by

   $r(A) \approx P_1(A) =$

   (d) The quadratic approximation for r as a function of A when A is near zero is given by

   $r(A) \approx P_2(A) =$

6. Solve these separable equations.

   (a) $u' = t^2\sqrt{u}$

   (b) Problem #7.2.15: $\sqrt{x}\dfrac{dy}{dx} = e^{y+\sqrt{x}}$

7. For each of these separable equations, say whether the solution grows without bound, approaches a finite limit as $x \to \infty$ or blows up ($y \to \infty$ at some finite value of x). If it depends on the initial conditions, please be sure to point this out. You do not have to solve the equation.

   (a)
   $$\frac{dy}{dx} = \frac{y^2}{1+x}$$

   (b)
   $$y' - \sqrt{1+y}e^{-x} = 0$$

   (c)
   $$y' = \frac{x^{3/2}}{1+y^2}$$

   (d)
   $$2yy' = \sqrt{x+x^5}$$

8. Solve these first order linear differential equations. You may need to introduce notation for the antiderivative of any function you are not able to integrate.

   (a) (Problem # 6 in Section 9.2):

   $$(1+x)y' + y = \sqrt{x}$$

   (b) $y' - \dfrac{y}{\ln t} = 1$ (you can assume $t > 1$)

9. (Problem # 15 in Section 9.2): Solve this first order linear initial value problem.

$$\frac{dy}{dt} + 2y = 3\,; y(0) = 1\,.$$

10. Solve this first order linear initial value problem. You may need to introduce notation for the antiderivative of any function you are not able to integrate.

$$y' - \frac{y}{\sqrt{1+x^4}} = 0\,;\; y(0) = 1\,.$$

11. (Problem # 19 in Section 9.2): Solve this first order linear initial value problem.

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = \frac{e^{x^2}}{x+1}\ ;\ y(0) = 5\,.$$