

Math 110, Spring 2015
HWK02 due WED 04 February

1. My grandma gave me a \$1000 savings bond when I was born, that earned 6% interest each year (it is not compounded during the year). Use logs to estimate (without a calculator) the present value (I am now 50). Please show all your work. [Note: you will have to decide what base to take logs in, and you will have to use linear estimates to estimate the logs.]

2. Define a function f on the interval $(1, \infty)$ by $f(x) = x^{1/\ln x}$. Sketch the graph of f and provide justification as to why your sketch looks the way it does. You may use a calculator to compute values of f but please do not use a graphing calculator to graph f .

3. Use L'Hôpital's rule to prove these facts about exponentials, logarithms and orders of growth.

(a) For any integer n , $x^n = o(e^x)$.

(b) For any integer n , $\ln x \ll o(x^{1/n})$.

4. Determine whether each statement is true or false.

(a) $\ln(2x) \sim \ln(x)$

(b) $e^{2x} \ll x e^x$

(c) $\sqrt{x^2 + 1} - x \asymp \frac{1}{2x}$

5. Famous story¹: In ancient times a Persian king wanted to reward a young man who had saved the life of his daughter. He pointed to the royal chessboard and offered the young man a choice. He could either have 100 gold coins on the first square, 200 on the second, 300 on the third, and so on, or else have one grain of wheat on the first square, two grains of wheat on the second square, four on the third, eight on the fourth, and so on. Which should the young man have chosen? Please use logs to estimate how many grains of wheat were on the last square or on the whole board. Make any reasonable assumptions about number of grains in a bushel, price of wheat, worth of gold, etc. Do not take literally that the gold or the wheat has to fit on the chessboard.

¹For one version, see <http://www.dr-mikes-math-games-for-kids.com/rice-and-chessboard.html> .

6. (a) Determine whether $2^x \ll x^2$, or vice versa or neither.

(b) Explain the relevance of this to the previous problem.