Units 12-13: Integrals over the whole real line and probability densities

Vocabulary and notation

Improper integral	DNE	undefined integral	\int^{∞}
probability density	probability	random variable	mean
exponential density	normal density	standard normal	uniform density
standard deviation	median	average value	normalizing constant
Φ	half life	convolution	

Skills

- Know the definition of an improper integral via limits
- Know for which k, p and q these integrals converge:

$$-\int_1^\infty e^{kx} dx$$

$$-\int_1^\infty x^p dx$$

$$-\int_1^\infty (\ln x)^q x^{-1} dx$$

- Know the relation between convergence of $\int_b^\infty f(x) \, dx$ and convergence of $\int_b^\infty g(x) \, dx$ when $f \ll g$ or $f \sim g$ as $x \to \infty$.
- Know how to find p so that $f(x) \sim cx^p$ as $x \to \infty$ when f is a more complicated function.
- Know the relation between convergence of series and convergence of integrals
- Know the exponential, uniform and normal densities
- Be able to compute the mean and median of the exponential
- Have an idea of when to use these distributions in modeling
- Know how to standardize a normal random variable (last sentence of the Unit)
- Know how to compute a convolution of two probability densities and what this means probabilistically