Units 12-13: Integrals over the whole real line and probability densities

Vocabulary and notation

<table>
<thead>
<tr>
<th>Improper integral</th>
<th>DNE</th>
<th>undefined integral</th>
<th>$\int^{\infty}$</th>
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</thead>
<tbody>
<tr>
<td>probability density</td>
<td>probability</td>
<td>random variable</td>
<td>mean</td>
</tr>
<tr>
<td>exponential density</td>
<td>normal density</td>
<td>standard normal</td>
<td>uniform density</td>
</tr>
<tr>
<td>standard deviation</td>
<td>median</td>
<td>average value</td>
<td>normalizing constant</td>
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<tr>
<td>$\Phi$</td>
<td>half life</td>
<td>convolution</td>
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</tbody>
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Skills

- Know the definition of an improper integral via limits
- Know for which $k, p$ and $q$ these integrals converge:
  - $\int_{1}^{\infty} e^{kx} \, dx$
  - $\int_{1}^{\infty} x^{p} \, dx$
  - $\int_{1}^{\infty} (\ln x)^{q} x^{-1} \, dx$
- Know the relation between convergence of $\int_{b}^{\infty} f(x) \, dx$ and convergence of $\int_{b}^{\infty} g(x) \, dx$ when $f \ll g$ or $f \sim g$ as $x \to \infty$.
- Know how to find $p$ so that $f(x) \sim cx^{p}$ as $x \to \infty$ when $f$ is a more complicated function.
- Know the relation between convergence of series and convergence of integrals
- Know the exponential, uniform and normal densities
- Be able to compute the mean and median of the exponential
- Have an idea of when to use these distributions in modeling
- Know how to standardize a normal random variable (last sentence of the Unit)
- Know how to compute a convolution of two probability densities and what this means probabilistically