

# 106: Geometry

## 1. Length

- Measurement
  - need for standard units
  - units and their comparison
  - successful measurement: alignment, no gaps, no overlapping
- Length and Distance
  - length of a curve
  - distance in cartesian coordinates
- Behavior under Scaling and Change of Units

## 2. Area

- Defining Area
  - conservation of area
  - comparing by dissecting and then rearranging pieces
  - areas of irregular shapes
  - units of area
  - surface area
- Area Formulae
  - rectangles: integer and non-integer
  - triangles: justification of  $\frac{1}{2}b \cot h$
  - justification of formulae for parallelogram, trapezoid, circle
  - applications: geoboards, word problems
- Behavior under Scaling and Change of Units
  - proportionality to the square of the scale factor
  - two-dimensionality
  - independence of area and perimeter

## 3. Volume

- Defining Volume (analogous to Defining Area, above)
- Prisms and Cylinders
  - product formulae
  - right versus oblique cylinders
  - pyramids and cones: the mysterious factor of  $\frac{1}{3}$
- Behavior under Scaling and Change of Units
  - proportionality to the cube of the scale factor
  - three-dimensionality
  - independence of volume and surface area

## 4. Angle

- Defining and Measuring Angles
  - using a protractor
  - alignment and orientation
  - units of angle
- Summing Angles
  - adding adjacent angles
  - summing angles of a triangle
  - summing interior angles of a polygon
- Behavior under Scaling and Change of Units
  - zero-dimensionality: invariance under scaling

- independence of angle from linear measure of subtended chord
- similarity
- applications: measuring from afar, surveying

## 5. Euclidean Geometry

- Axiomatic Reasoning
  - Euclid’s axioms
  - rigidity: intuition behind the congruence axioms
  - proofs of some simple facts
  - motivation for euclidean rigor: false informal proofs
- Constructions with Straightedge and Compass
  - relation to the axioms
  - basic constructions: copying, bisecting, erecting perpendiculars
  - proving your construction works
- New Proofs of Old Formulae
  - rigorizing proof of area formulae
  - pythagorean theorem

## 6. Symmetry and Rigid Motion

- Rigid Motions
  - reflections, rotations, translations
  - composing rigid motions
- Symmetry of Plane Figures
  - reflections and lines of symmetry
  - rotations and centers of symmetry
  - finding all the symmetries
- Tessellations and Their Symmetries
  - translational symmetry
  - wallpaper patterns
  - classification of regular tessellations
  - classification of semi-regular tessellations
  - symmetries of tessellations
- Three-Dimensional Shapes
  - Platonic solids
  - planes of reflection
  - axes of rotation
  - symmetries of regular polyhedra and other polyhedra

## 7. General Skills

- Spatial imagination
  - nets for folding polyhedra
  - visualizing solids in other orientations
  - counting faces, edges and vertices
  - two-dimensional projections
- Drawing and Sketching
- Vocabulary
  - names of shapes: circle, trapezoid, tetrahedron, etc.
  - components: vertices, edges, lines, points, rays, etc.
  - components of figures: vertical angles, parallel lines, similar triangles, etc.
  - properties of shapes: isosceles, regular, right, congruent, etc.
  - concepts: area, rigid motion, polygon, etc.
- Broader Context: Role of Geometric Intuition in Mathematical Concept Formation