# Model solutions to more word problems

Problems 1 and 4 illustrated the when you have a quantity that changes, you need to use different variables for the quantity at different times.

**1. Oldest and youngest.**

- *Variables:*
  - $X$ is the age of my oldest child right now
  - $Y$ is the age of my youngest child right now
  - $A$ is the age of my oldest child next year
  - $B$ is the age of my youngest child next year
- *Equations:*

  $$X = 4Y$$

  It is given that the oldest is four times as old as the youngest right now.

  $$A = 3B$$

  It is given that the oldest will be three times as old as the youngest next year.

  $$A = X + 1$$

  The oldest child's age must increase by exactly 1 in one year.

  $$B = Y + 1$$

  The youngest child's age must increase by exactly 1 in one year.
- *Answer:* Solving these yields $X = 8, Y = 2, A = 9, B = 3$; thus there ages right now are 8 and 2.

**1a. Oldest and youngest (alternate solution).**

- *Variables:*
  - $X$ is the age of my oldest child right now
  - $Y$ is the age of my youngest child right now
- *Equations:*

  $$X = 4Y$$

  It is given that the oldest is four times as old as the youngest right now.

  $$(X + 1) = 3(Y + 1)$$

  It is given that the oldest will be three times as old as the youngest next year.
- *Answer:* Solving these yields $X = 8, Y = 2$; thus there ages right now are 8 and 2.

NOTE: In the second solution, we did not incorporate new variables for the ages next year, but rather used expressions for those quantities $(X + 1)$ and $(Y + 1)$ in terms of the variables representing the ages this year. Nevertheless, it is still important to label the variables correctly as pertaining to ages *this year*: otherwise, it gets very confusing and very difficult to see where all the equations are hiding.

**4. State Senate.**

- *Variables:*
  - $r$ is the number of Republicans in the senate now.
  - $d$ is the number of Democrats in the senate now.
- *Equations:*

  $$r = 2d$$

  This is what a two to one ratio means.

  $$(d + 5) = (r - 5) + 1$$

  According to the problem, $d - 5$ and $r + 5$ represent respectively the number of Democrats and the number of Republicans in the senate next year. If the Democrats hold a one-seat advantage after the elections, then $d + 5$ must be one more than $d - 5$.

NOTE: Just as with problem 1, you can solve this by adding more variables (e.g., let $x$ be the number of Republicans in the senate after the 2002 elections and $Y$ be the number of Democrats in the senate after the 2002 elections). The method will not work as well when not all the quantities can be written in terms of the one quantity initially guessed. You can still use the method in this case, but will need multiple guess variables and multiple equations at the end, so it is less likely to help you see when to guess and when to use an expression in terms of previous variables.

In problems 2 and 3, I will illustrate a means of solution I'll call "guess and check, with teeth". This can be helpful when you are able to visualize the problem but are having trouble finding equations.

**3. Van and Cruiser.**
Suppose we try guess and check, guessing (as seems reasonable) that she drives 300 miles in the Cruiser (you will see it doesn't matter which guess you try first). Could this be right? What would her total mileage be in this case? To answer these questions, we compute:

- She drives 300 miles in the Cruiser.
- Therefore she drives 300 miles in the Van.
- She uses 15 gallons of gas in the Cruiser.
- She uses 30 gallons of gas in the Van.
- Therefore, her total gas consumption is 45 gallons.
- Therefore her total mileage is 13 1/3 MPG.
- Therefore this was the wrong guess.

If you could not get this far with your guess, then you had trouble visualizing the problem. Suppose you did get this far. The next step is to guess a variable instead of the initial guess of 300. You then replicate the computation with a variable guess. NOTE: you can save time and guess a variable right off the bat, but often it is too difficult to see what is happening to the variables without having had a numerical example to guide you. To clarify the role of the numerical guess, I will list the thought processes as I go (not something normally done on a write-up). Each step of the numerical solution has a corresponding algebraic step, after which I give the thinking that led to the step.

- She drives $X$ miles in the cruiser. *Reason:* the method says, choose a variable as a value for the initial guess.
- Therefore she drives $600-X$ miles in the Van. *Reason:* I wasn't sure at first how to write the number of miles driven in the van, but after trying a few examples (400 miles in the Cruiser gives 200 miles in the Van, and so on) I realized how to make explicit the relationship that I understood implicitly.
- She uses $X/20$ gallons of gas in the Cruiser. *Reason:* Once I had to figure out how I got the number 15 in the numerical example, I realized I had divided 300 miles by 20 MPG. The units miles, divided by miles per gallon, come out to gallons, which corroborates that I have represented this relationship correctly.
- She uses $(600 - X)/10$ gallons in the Van. *Reason:* I never would have gotten this had I tackled it head-on, but with the numerical solution in front of me I could see the analogy to the computation of the number of gallons used in the Cruiser.
- Therefore, her total gas consumption is

  $$\frac{X}{20} + \frac{600 - X}{10}$$

  gallons for the week. *Reason:* It was obvious that the 45 came from $15 + 30$, so I just checked that this was a sum that made sense, and indeed, gallons plus gallons yields gallons.
- Therefore her total mileage was

  $$\frac{600}{\frac{X}{20} + \frac{600-X}{10}}$$

  miles per gallon. This looks ugly, but I have to believe it is true. After all, the 13 1/3 came from 600 total miles divided by 45 gallons, and I am just replicating this computation with variables.
- Therefore, my guess was right if and only if

  $$15 = \frac{600}{\frac{X}{20} + \frac{600-X}{10}}\,.$$

  *Reason:* She is supposed to get 15 miles to the gallon, so if I ask that the total miles in units of miles per gallon, which I just computed, be equal to 15, I must be writing an equation satisfied by the problem. Since there is only one variable the way I did it, this is the only equation I need to write. Solving it will give me $X$ and solve the problem.

Answer: solving yields $X = 400$, so she drives 400 miles in the Cruiser and 200 miles in the van.

**2. GPA**

Often you'll be able to do the computations for the numerical guess without supplying reasons. Then, when you go back and do it with algebra, you'll need to see what the reasons were in order to get the algebra right. In other words, you let the numerical problem you already did guide to to the algebraic solution.

- Guess: Maybe Professor Nice gives 60% of the students A's.
- Therefore 30% get B's.
- The GPA for the class is 3.3.
- The GPA was supposed to be 3.5 so my guess was wrong.

Now we do it with algebra.

- Guess: $X\%$ of the students get A's.
- How many get B's? Where did the 30% come from? I saw that 60% were accounted for with A's and 10% failed, which left 30%. Algebraically, if I replace the 60% by X%, the 10% won't change, and to find what is left I have to subtract both the X% and the 10% from 100%. Now instead of leaving 30%, it leaves $(90 - X)\%$.
- What is the GPA? I have to see how I got the 3.3 last time. I computed the total number of grade points and divided by the number of students. Actually, I didn't know the totals, so I plugged in percentages instead:
$$\frac{60\% \times 4 + 30\% \times 3 + 10\% \times 0}{100\%}.$$
With a guess of $X\%$, this becomes:
$$\frac{X\% \times 4 + (90 - X)\% \times 3 + 10\% \times 0}{100\%}$$
which comes out to $(270 + X\%)/100\%$.
- We know the GPA is supposed to be 3.5, so $(270 + X)\%/100\% = 3.5$.

Answer: Therefore $270 + X = 350$ and $X = 80$, so 80% got A's.

Now do these two problems, using the two methods you have just learned. For the first problem, make sure you identify variables representing a single value. For the second, use guess and check with teeth.

1. A study showed that one out of every eleven people born in Ohio possesses a gene for depression. Later it was found that one of the subjects was not from Ohio so her data was deleted from the file. Now the study shows that one in every twelve people born in Ohio possesses this gene. How many people were in the study?

2. With no wind, a Commercial Jet can average 500 MPH, getting to Denver and back in 4 hours. Any headwind slows it down in one direction (by whatever the wind speed is) and speeds it up in the other (by the same amount). How many MPH of headwind would it take to increase the round-trip flying time by 10 minutes?