

105: Arithmetic

1. Foundations

- Sets
 - what is a set: notation, extension property
 - subsets, unions, intersections, complements
 - Venn diagrams (see also Counting: inclusion-exclusion, below)
- Logic
 - informal logic: parsing propositional statements
 - formal logic: and, or, not, if-then
 - truth tables and deduction
- Operations (see Whole Numbers: Properties of Operations, below)

2. Whole Numbers and their Properties

- Counting
 - one to one correspondence
 - inclusion-exclusion
 - cartesian products
- Factors
 - prime numbers
 - unique prime factorization
 - counting factors
 - LCM: how to obtain it; properties of the LCM
 - GCD: how to obtain it; properties of the GCD
- Negative Numbers
 - interpretation of negative numbers
 - rules for manipulation of negative numbers
- Properties of Operations
 - abstract operations: commutativity, associativity, identities, inverses
 - properties of addition, multiplication, subtraction, division
 - consequences of these properties
 - distributivity
 - mental math consequences
- Place Value
 - how our numeration system works
 - other bases
 - algebraic representation of numbers in terms of their digits
 - negative place value and decimals (see Decimals: place value, below)
- Common Algorithms
 - justification of addition, subtraction and multiplication algorithms
 - understanding alternative algorithms

3. Division and Fractions

- Division
 - inverse of multiplication
 - repeated subtraction
 - division algorithms
 - division of fractions (see: Fractions: multiplication and division, below)
- Ratio and Proportion
 - proportionality word problems
 - recognizing when given data is in terms of proportion

- cross-multiplication and its justification
- Fractions
 - models of fractions: number line/measurement, ratio, quotient, operator
 - equivalent fractions and cross-multiplication
 - mixed versus improper fractions
 - adding and subtracting fractions
 - multiplication and division: models, procedures
- Decimals
 - place value
 - terminating decimals to fractions
 - classification of fractions as terminating or repeating decimals
- Scientific Notation
 - definition and conversion into and out of
 - estimation of physical quantities
 - approximate computation in scientific notation
 - logarithmic scales: brief intro
- Percentages
 - representation as a fraction or decimal
 - percentage increase and decrease

4. General Skills and Attitudes

- Problem Solving Strategies
 - trial and error
 - reducing the problem
 - numbers instead of variables
 - harnessing physical intuition
- Generalization and Abstraction
 - observing patterns
 - expressing your observations
 - using variables to express and justify observations
 - broadening the scope
- Word Problems
 - understanding what is asked
 - unambiguous usage of variables
 - units
 - writing equations for a problem
- Broader Context
 - NCTM Standards for active learning, justification/proof, verbalizing
 - articulate and grammatical mathematical expression
 - argumentation