Volumes Using Cylindrical Shells

Snapshot

- **Major Concept:** The volume of rotationally-symmetric three-dimensional regions can also be expressed as an integral of surface areas of cylindrical shells centered on the symmetry axis.
- **Before You Begin:** Recall formulas for the surface area of circular cylinders.
- **Standards for Practice and Evaluation:** Use the "shell method" to compute volumes of regions. Pay particular attention to regions where the axis is in an unusual direction (any of the three coordinate directions) and/or an unusual position. For any given problem, be able to **quickly** determine which of the disk, washer, or shell methods will be easiest to apply.

Worksheet Objective

In this worksheet, you will be asked to describe and demonstrate the process involved in computing volumes using the shell method. You will also be asked to spend some time comparing and contrasting the methods of Sections 6.1 and 6.2 and practicing problems in which you are responsible for choosing the best methods.

Volumes: The Shell Method

Remember **Understand** Apply Analyze Evaluate Create

Section 6.2 begins with a derivation of the formulas for the shell method. In your own words, where does the formula come from? In particular, what is being "sliced"? What are the areas and volumes involved? What role do finite sums play, and how do we get to the final integral formula? Draw pictures to illustrate your ideas.

Remember **Understand** Apply Analyze Evaluate Create

Section 6.2 ends with a three-step summary of what is involved in applying the shell method. In your own words, what are the steps?

Remember | Understand | **Apply** | Analyze | Evaluate | Create

(6.2.24f) Use the shell method to find the volume of the solid generated by revolving the region bounded by the curves

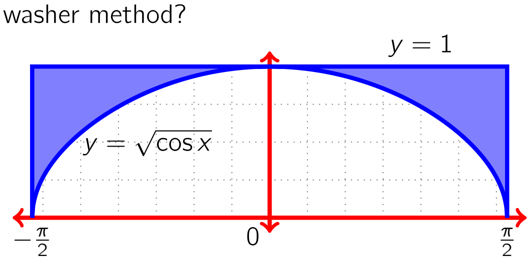
$$y = x^3, \ y = 8, \ x = 0$$

about the line $y = -1$. Find the volume.

Comparison of Disks, Washers, and Shells

Remember | Understand | Apply | **Analyze** | Evaluate | Create

(6.1.37) Use the shell method to write an integral whose value equals the volume of the solid generated by revolving the shaded region about the x-axis. **DO NOT EVALUATE THE INTEGRAL.** How does the complexity compare to the washer method?



Remember | Understand | Apply | **Analyze** | Evaluate | Create

The region in the plane bounded by the curves $x = 2 - y^2$, $x = 1$, and $y = 0$ is revolved around the y-axis. Compute the volume using whichever method you prefer.

Remember | Understand | Apply | **Analyze** | Evaluate | Create

(6.2.42) A Bundt cake, well-known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and the x-axis over the interval $1 \leq x \leq \sqrt{1 + \pi}$. Find the volume of the cake. Use whichever method you prefer.

Remember | Understand | Apply | Analyze | **Evaluate** | Create

What are some criteria to use when deciding which of the three methods to apply to a particular problem?

Remember | Understand | Apply | Analyze | Evaluate | **Create**

For each method, draw a picture and construct a region for which the indicated method would likely be the easiest to apply.

The Disk Method	The Washer Method	The Shell Method

Review and Summary

Practice these skills to prepare for future evaluation:

- When asked to compute the volume of a solid, quickly determine which of these three methods (if any!) is best.
- If one of the methods applies, set up the integral carefully, being aware of and avoiding common mistakes (using the wrong axis, radius, endpoints, etc.)

Read sections 6.3 and 6.4. The topics are a departure from volume calculation, but continue on the theme of geometric quantities which can be computed by integration.