2

Using the quotient rule,

$$\begin{aligned}\tanh'(x) &= \frac{(e^x+e^{-x})(e^x+e^x)-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}\\ &= \frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2}-\frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2}\\ &= 1-\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2\\ &= 1-(\tanh x)^2.\end{aligned}$$

Therefore

$$\tanh'(\tanh^{-1} x) = 1-(\tanh(\tanh^{-1} x))^2 = 1-x^2.$$

Therefore using the rule for the derivative of an inverse function,

$$\frac{d}{dx}\tanh^{-1} x = \frac{1}{\tanh'(\tanh^{-1} x)} = \frac{1}{1-x^2}.$$

3

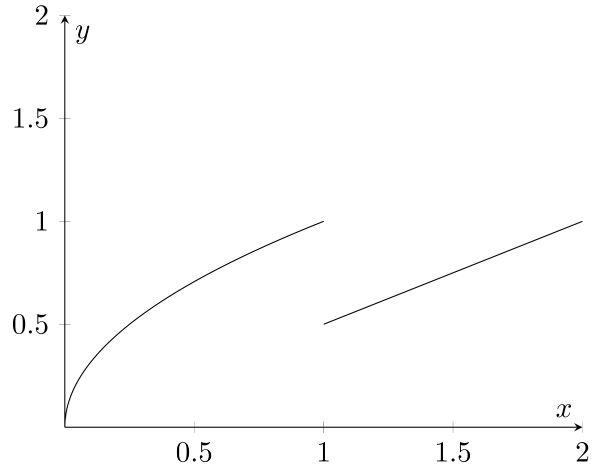
In order for these piecewise functions to be differentiable at 1, two things need to happen:

1. The two pieces need to meet—that is, it needs to be continuous at 1, so the two parts need to have the same value at 1,
2. The pieces need to meet smoothly—the derivatives of the two parts need to have the same value at 1.

3a

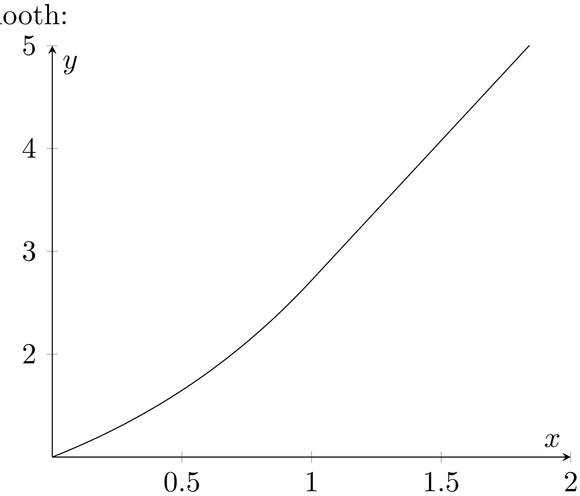
$\sqrt{1} = 1 \neq 1/2$, so this is not continuous at 1, so it cannot be differentiable at 1.

A picture makes pretty clear what goes wrong:



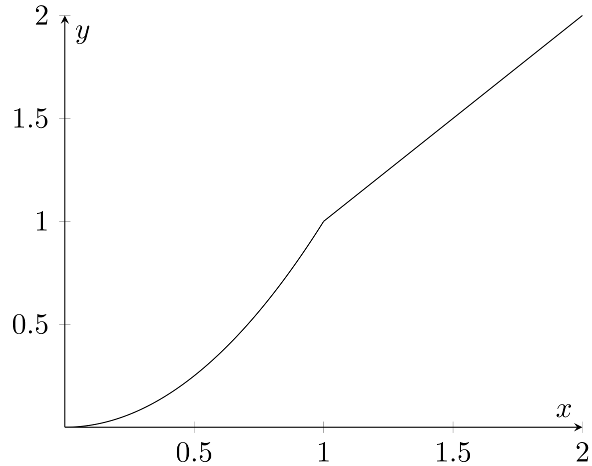
3b

$e^1 = e = e \cdot 1$, so the two pieces meet at 1. The derivatives are e^x and e, which again agree at 1. In the picture, we can see that the function looks smooth:



3c

$1^2 = 1$, so the pieces meet at 1. The derivatives are $2x$ and 1, so at 1 the derivatives are $2 \cdot 1 \neq 1$, so the function is not differentiable. In the picture we see that though there isn't a jump (the function is continuous), there is a sharp corner:



5

5a

The units of $Q(p)$ will be the number of units sold. The units of p will be a currency, for instance, dollars. The units of $Q'(p)$ will be units per dollar. For most goods, $Q'(p)$ is negative: if we raise the price, we sell less.

5b

The units are

$$\frac{\text{dollars}}{\text{units}} \cdot \frac{\text{units}}{\text{dollars}}$$

so $E(p)$ is unitless (it has no units).

5c

Since $\ln Q(p) = \ln 3p^{-1/2} = \ln 3 + \ln p^{-1/2} = \ln 3 - \frac{1}{2}\ln p$, $[\ln Q(p)]' = -\frac{1}{2p}$.

5d

$E(p) = p\frac{Q'(p)}{Q(p)} = p[\ln Q(p)]' = p(-\frac{1}{2p}) = -\frac{1}{2}$.

6

We have $d = \frac{v^2 \sin 2\theta}{g}$. This problem asks us for to consider two situations.

First situation

We treat v as constantly equal to 50 and θ as a variable. We want Δd when $\Delta\theta = 0.2$; we use the approximation formula for error: $\Delta d \approx d'(\theta_0)\Delta\theta$ (where we take the derivative with respect to θ). We have $d(\theta) = \frac{v^2 \sin 2\theta}{g} = \frac{50^2 \sin 2\theta}{g}$ (because v is constantly 50), so $d'(\theta) = \frac{2 \cdot 50^2 \cos 2\theta}{g}$, so

$$\Delta d \approx d'(\theta_0)\Delta\theta = \frac{2 \cdot 50^2 \cos 2\theta_0}{g}\Delta\theta = \frac{2 \cdot 50^2 \cos \pi/4}{g}\Delta\theta = 0$$

because $\theta_0 = \pi/4$.

Second situation

We treat θ as constantly equal to $\pi/4$ and v as a variable. We want Δd when $\Delta v = 0.2$; we use the approximation formula for error, $\Delta d \approx d'(v_0)\Delta v$ (where we take the derivative with respect to v). We have $d(v) = \frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin(\pi/2)}{g} = \frac{v^2}{g}$, so $d'(v) = \frac{2v}{g}$. So

$$\Delta d \approx d'(v_0)\Delta v = \frac{2v_0}{g}\Delta v = \frac{100}{g}\Delta v = \frac{100}{g} \cdot 0.2.$$

So the error when we get v wrong is much bigger.