a

Working with an Example

Like most groups, I'll take $g(t) = t^2$ as my example function. So after t seconds, the car has travelled t^2 miles.

1 hour is 3600 seconds, so after 1 hour the car has travelled $(3600)^2$ miles. The distance travelled after 1 hour is $h(1)$, so

$$h(1) = (3600)^2.$$

After 2 hours we've travelled for $2 \cdot 3600 = 7200$ seconds, so we've travelled $(7200)^2$ miles. The distance travelled after 2 hours is $h(2)$, so

$$h(2) = (7200)^2 = (3600 \cdot 2)^2.$$

After 1.5 hours we've travelled for $1.5 \cdot 3600 = 5400$ seconds, so we've travelled $(5400)^2$ miles. The distance travelled after 1.5 hours is $h(1.5)$, so

$$h(1.5) = (5400)^2 = (3600 \cdot 1.5)^2.$$

We notice the pattern

$$h(t) = (3600 \cdot t)^2.$$

We can rewrite this in terms of g:

$$h(t) = (3600t)^2 = g(3600t).$$

For an Abstract g

The relationship between h and g in the example is pretty compelling. Indeed, after t hours, we have travelled for $3600t$ seconds, so we've travelled $g(3600t)$ miles. Therefore

$$h(t) = g(3600t).$$

b

Working with an Example

Let's take $g'(t) = 2t$ to be our example function. So when we've been travelling for t seconds, our velocity is $2t$ miles/second. After travelling for

1 hour, we've travelled for 3600 seconds, so our velocity is $2 \cdot 3600 = 7200$ miles/second. In order to convert this into miles/hour, we multiply by 3600:

$$h'(1) = 3600(2 \cdot 3600) = 3600 \cdot 7200 = 3600 \cdot 2 \cdot 3600 \text{ miles/hour.}$$

When we've been travelling for 2 hours, we've travelled for 7200 seconds, so our velocity is $2 \cdot 7200 = 14400$ miles/second, so

$$h'(2) = 3600 \cdot (14400) = 3600 \cdot 2 \cdot 7200 \text{ miles/hour.}$$

We see the pattern

$$h'(t) = 3600 \cdot 2 \cdot 3600t,$$

and notice that this is the same as

$$h'(t) = 3600g'(3600t).$$

For an Abstract g

We again hope that the same relationship might hold. Indeed, whatever g is, $g'(t)$ is velocity after t seconds in miles/second, so $g'(3600t)$ is velocity after t *hours* in miles/second, so $3600g'(3600t)$ is velocity after t hours in miles/hour, so

$$h'(t) = 3600g'(3600t).$$

C

In order to find $h'(1)$, the velocity at 1 hour, we need to know $g'(3600)$, the speed at 3600 seconds.