Definition 1. A **Semistandart Young Tableau (SSYT)** of shape λ is a filling of the Young diagram of λ with positive integers, such that rows are weakly increasing to the right and columns are strictly increasing top to bottom. The **type** of an SSYT T is $\mu = (1^{m_1}, 2^{m_2}, \ldots)$, where m_i is the number of is appearing in T. A Standard Young Tableau (SYT) of shape λ is an SSYT of shape $\lambda \vdash n$ and type (1^n).

Examples: SSYT T of shape $\lambda = (4, 3, 2, 2)$, $T =$

1	1	3	4
2	4	5	
4	5		
6	6		

.

$type(T) = (1^2, 2, 3, 4^3, 5^2, 6^2)$.

SYT T of shape $\lambda = (3, 2, 1, 1)$ $(|\lambda| = 7)$, $T =$

1	3	4
2	6	
5		
7		

.

Definition 2. The **Schur function** of $x = (x_1, x_2, \ldots)$, corresponding to partition λ, denoted $s_\lambda(x)$ is defined as

$$s_\lambda(x) = \sum_{T:SSYT,\mathrm{sh}(T)=\lambda} x^{type(T)},$$

where the sum runs over all SSYT of shape λ and $x^{type(T)} = x_1^{m_1} x_2^{m_2} \ldots$, where $type(T) = (1^{m_1}, 2^{m_2}, \ldots)$.

Example:

$$s_{(2)}(x_1, x_2, x_3) = x_1^2 + x_1x_2 + x_2^2 + x_1x_3 + x_2x_3 + x_3^2 = h_2(x),$$

$$s_{(1^n)}(x) = \sum_{i_1<i_2<\cdots<i_n} x_{i_1} \ldots x_{i_n} = e_n(x)$$

(here $T =$ $\begin{array}{|c|}\hline i_1 \\ \hline i_2 \\ \hline \vdots \\ \hline i_n \\ \hline\end{array}$)

$$s_{(2,2)}(x_1, x_2) = x_1^2 x_2^2, s_{(2,2)}(x_2, x_3, x_4) = x_2^2 x_3^2 + x_2^2 x_4^2 + x_3^2 x_4^2 + x_2^2 x_3 x_4 + x_2 x_3^2 x_4 + x_2 x_3 x_4^2.$$

Definition 3. Row insertion of k into an SSYT T, denoted $(T \leftarrow k)$ is the procedure outlined as follows: Find the first element T_{1i} in the first row of T, such that $T_{1i} > k$, then replace T_{1i} by k and insert T_{1i} into the next row of T the same way, until at step j there is no T_{ji} larger than the currently inserted element k', at which point we put k' at the end of the row.

Definition 4. The **RSK** algorithm is a map between matrices A with nonnegative integer entries (finitely many nonzero) and pairs of same shape SSYTs (P, Q), outlined as follows: $A \to w_A$, where w_A is the lexicographically sorted two-row sequence, consisting of $\begin{smallmatrix} i \\ j \end{smallmatrix}$, each appearing a_{ij} times. $(P_0, Q_0) = \emptyset$, step k: let the kth column of w_A be $\begin{smallmatrix} i_k \\ j_k \end{smallmatrix}$, then $P_k := (P_{k-1} \leftarrow j_k)$ and $Q_k = Q_{k-1} \bigcup i_k$, where i_k is put at the place where the new box was added in P_k.

Theorem 1. RSK is a bijection.

Corollary 1.

$$\prod_{i,j} \frac{1}{(1 - x_i y_j)} = \sum_\lambda s_\lambda(x) s_\lambda(y).$$

Example of RSK (and row insertion at each step, the insertion paths are underlined),

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow w_A = \begin{pmatrix} 1 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 3 \\ 3 & 4 & 5 & 5 & 1 & 3 & 1 & 1 & 5 \end{pmatrix}$$

P	Q
$\underline{\mathbf{3}}$	$\mathbf{1}$
3 $\underline{\mathbf{4}}$	1 $\mathbf{1}$
3 4 $\underline{\mathbf{5}}$	1 1 $\mathbf{1}$
3 4 5 $\underline{\mathbf{5}}$	1 1 1 $\mathbf{1}$
$\underline{1}$ 4 5 5 $\underline{\mathbf{3}}$	1 1 1 1 $\mathbf{2}$
1 $\underline{3}$ 5 5 3 $\underline{\mathbf{4}}$	1 1 1 1 2 $\mathbf{2}$
1 $\underline{1}$ 5 5 3 $\underline{3}$ $\underline{\mathbf{4}}$	1 1 1 1 2 2 $\mathbf{3}$
1 1 $\underline{1}$ 5 3 3 $\underline{\mathbf{5}}$ 4	1 1 1 1 2 2 $\mathbf{3}$ 3
1 1 1 5 $\underline{\mathbf{5}}$ 3 3 5 4	1 1 1 1 $\mathbf{3}$ 2 2 3 3

Definition 5. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be a composition (i.e. no order restrictions), define $a_\alpha(x_1, \ldots, x_n) := \det[x_i^{\alpha_j}]_{i,j=1}^n$. Set $\delta = (n-1, n-2, \ldots, 1, 0)$, $\lambda + \delta = (\lambda_1 + n - 1, \lambda_2 + n - 2, \ldots, \lambda_n)$.

Theorem 2.

$$s_\lambda(x_1, \ldots, x_n) = \frac{a_{\lambda+\delta}}{a_\delta}.$$

Theorem 3. (Jacobi-Trudi identity)

$$s_{\lambda/\mu} = \det[h_{\lambda_i - i - (\mu_j - j)}]_{i,j=1}^n,$$

where $n = l(\lambda)$.