

The final project should be a 7-10 page expository paper on topic related to symmetric functions. The paper is due 2 days prior to the end of reading period, **May 4** (this is sharp - the grades for the class are due May 6). The papers will be presented in the last 3 classes (40 minute presentations).

Here is a list of sample topics:

- Representations of the Symmetric Group and/or the General Linear Group. Sources: EC2, 7.18 and appendix 2; Young tableaux: with applications to representation theory and geometry, William Fulton; and many more... Notes on representation theory: <http://math.berkeley.edu/~serganov/math252/index.html>
- Enumeration of Standard Young Tableaux, including the various proofs of the hook-length formula. Sources: C.Krattenthaler, 'Bijective proofs of the hook formulas for the number of standard Young tableaux, ordinary and shifted ', *Electron. J. Combinatorics*; Novelli, Pak, Stoyanovskii, 'A new proof of the hook-length formula ', *Discrete Math. and Theoretical Computer Sci.*; B.E.Sagan, 'The ubiquitous Young tableau', *Invariant Theory and tableaux*; Greene, Nijenhuis, Wilf, 'A probabilistic proof of a formula for the number of Young tableaux of a given shape', *Advances in Math.*
- The Littlewood-Richardson rule - a combinatorial interpretation of the numbers  $c_{\mu\nu}^{\lambda}$ . Sources: EC2, appendix 1 (available at <http://www.math.lsa.umich.edu/~fomin/Papers/app1.ps>); Fomin, Greene, 'A Littlewood-Richardson miscellany ', *European Journal of Combinatorics*
- Symmetric functions associated to graphs - generalizations of the chromatic polynomial of the graph. Sources: R.Stanley, 'A symmetric function generalization of the chromatic polynomial of a graph', *Advances in Math.*; <http://www-math.mit.edu/~rstan/papers/taor.ps>
- Shifted Schur functions - Schur-type functions associated to shifted Young Tableaux (as in exercise 34). Sources: M.Haiman, 'On mixed insertion, symmetry and shifted Young tableaux', *J. Combinatorial Theory (A)*; Bruce E. Sagan, 'Shifted tableaux, schur Q-functions, and a conjecture of R. Stanley', *J. Comb. Theory, Ser. A* 45(1) etc.
- Quasisymmetric functions - a broader class of functions, closely related to posets ( $P$ -partitions), permutations and Schur functions. Sources: EC2, 7.19; Gessel, 'Multipartite  $P$ -partitions and inner products of skew Schur functions', *Contemp. Math.*
- Plane partitions - various enumerative results following from the theory of symmetric functions. Sources: EC2, 7.20,21,22; G.Andrews, 'Plane partitions (I)'; papers by R.Stanley, Hillam and Grassl, etc.
- Schubert polynomials - the connection between symmetric functions and algebraic geometry through Schubert calculus on the Grassmannian. Sources: <http://www.math.lsa.umich.edu/~fomin/Papers/balanced.ps>; and other papers by Sergey Fomin; Macdonald, Schubert calculus.

You are also welcome to come up with your own topic, a good source for ideas are the "exercises" in Richard Stanley's EC2 (Enumerative Combinatorics, Volume 2), Chapter 7 and the supplementary problems to EC2 (<http://math.mit.edu/~rstan/ec/ch7supp.pdf>) which are rated [3] or higher.