Math 21b, TTh 11:30 Section, Lecture 10, In Class Exercise
Orthonormal bases and orthogonal projections

Let V be the plane in $\mathbb{R}^3$ given by $x + y + 2z = 0$. Let $\vec{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$. Let $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. We want to find $\text{Proj}_V(\vec{x})$, the orthogonal projection of $\vec{x}$ onto the plane V.

- Check whether $\vec{v_1}$ and $\vec{v_2}$ are in the plane V.

- Next, check whether $\vec{v_1}$ and $\vec{v_2}$ are orthogonal, i.e. is the dot product $\vec{v_1} \cdot \vec{v_2}$ 0?

- Compute their lengths, $\|\vec{v_1}\| = \sqrt{\vec{v_1} \cdot \vec{v_1}}$ and $\|\vec{v_2}\| = \sqrt{\vec{v_2} \cdot \vec{v_2}}$.

- Find the orthonormal vectors $\vec{u_1} = \frac{1}{\|\vec{v_1}\|}\vec{v_1}$ and $\vec{u_2} = \frac{1}{\|\vec{v_2}\|}\vec{v_2}$.

- Convince yourself that $\vec{u_1}$ and $\vec{u_2}$ for an orthonormal basis for V and find the projection of $\vec{x}$ into V by the formula:

$$\text{Proj}_V(\vec{x}) = (\vec{x} \cdot \vec{u_1})\vec{u_1} + (\vec{x} \cdot \vec{u_2})\vec{u_2}.$$