Math 21b, Section 7, In-class exercise, SOLUTIONS
Fourier series

Find the Fourier series of the function

$$f(x) = |x|$$

on $[-\pi, \pi]$.

Is f even, odd (or neither)? even

Are any of the Fourier coefficients going to be 0 then? the sin series coefficientts will be 0.

Compute the Fourier series coefficients for $f(x) = |x|$:

$$a_0 =< f(x), \frac{1}{\sqrt{2}} >= \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\frac{1}{\sqrt{2}}dx = \frac{2}{\pi\sqrt{2}}\int_0^{\pi} xdx = \frac{2}{\pi\sqrt{2}}\frac{x^2}{2}\Big|_0^{\pi} = \frac{\pi}{\sqrt{2}}$$

$$a_n =< f(x), \cos(nx) >= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx =$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos(nx)dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\cos(nx)dx + \frac{1}{\pi}\int_0^{\pi} x\cos(nx)dx =$$

$$= 2\frac{1}{\pi}\int_0^{\pi} x\cos(nx)dx = 2\frac{1}{\pi}\int_0^{\pi} xd\frac{\sin(nx)}{n} =$$

$$2\ \frac{1}{\pi}x\frac{\sin(nx)}{n}\Big|_0^{\pi} - 2\frac{1}{\pi}\int_0^{\pi}\frac{\sin(nx)}{n}dx = 0 - 2\frac{1}{\pi}\frac{-\cos(nx)}{n^2}\Big|_0^{\pi} =$$

$$\frac{2}{\pi n^2}((-1)^n - 1) = \begin{cases} 0, & n \text{ - even,} \\ \frac{-4}{\pi n^2}, & n \text{ - odd} \end{cases}$$

$$b_n =< f(x), \sin(nx) >= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\sin(nx)dx$$

$$= 0,$$

since it is an integral of $|x|\sin(nx)$ - an odd function in an interval symmetric about 0.