Math 21b, Section 7, In-class exercise
Fourier series

Find the Fourier series of the function

$$f(x) = |x|$$

on $[-\pi, \pi]$.

Is f even, odd (or neither)?

Are any of the Fourier coefficients going to be 0 then?

Compute the Fourier series for $f(x) = |x|$:

$$a_0 =< f(x), \frac{1}{\sqrt{2}} >= \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \frac{1}{\sqrt{2}} dx = ...$$

$$a_n =< f(x), \cos(nx) >= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx =$$
$$\frac{1}{\pi} \int_{-\pi}^{\pi} |x| \cos(nx) dx = \frac{1}{\pi} \int_{-\pi}^{0} -x \cos(nx) dx + \frac{1}{\pi} \int_{0}^{\pi} x \cos(nx) dx =$$
$$= ...$$

$$b_n =< f(x), \sin(nx) >= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \sin(nx) dx$$
$$= ...$$