Math 21b, TTh 11:30 Section, Lecture 5, In Class Exercise
Image and Kernel, Error-correcting codes

We will try out the error-correcting code Hamming(7,4), which appears in exercises 53 and 54 *your homework!) of Section 3.1.

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1) Encoding. This is an encoding table for the 4-bits, i..e numbers in base 16:

$$\begin{array}{cccc} '0' = (0,0,0,0) & '1' = (0,0,0,1) & '2' = (0,0,1,0) & '3' = (0,0,1,1) \\ '4' = (0,1,0,0) & '5' = (0,1,0,1) & '6' = (0,1,1,0) & '7' = (0,1,1,1) \\ '8' = (1,0,0,0) & '9' = (1,0,0,1) & 'A' = (1,0,1,0) & 'B' = (1,0,1,1) \\ 'C' = (1,1,0,0) & 'D' = (1,1,0,1) & 'E' = (1,1,1,0) & 'F' = (1,1,1,1) \end{array}$$

Choose one of the symbols '1','2','3',...,'9','A','B','C','D','E','F' (no '0', please!). Then lookup its encoding vector $x = (d_1, d_2, d_3, d_4)$ from the table above and compute the encoding $(p_1, p_2, p_3, d_1, d_2, d_3, d_4)$ as the product $M.x$:

$$\underbrace{\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M} . \underbrace{\begin{bmatrix} -- \\ -- \\ -- \\ -- \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -- \\ -- \\ -- \\ -- \\ -- \\ -- \\ -- \end{bmatrix}}_{Mx} \tag{1}$$

2)Transmission. Now tell your neighbour the reulsting Mx, but don't be perfect! - introduce one error, i.e. change a 0 to 1, or vice versa at exactly one place in your vector. Respectively, you will receive some faulty information from your neighbour, it should be his Mx' (if his initial vector was $x' = (d_1', d_2', d_3', d_4')$), but it will differ by a standard vector e_i (a 1 in ith place) from his Mx'. Write it down:

$$\begin{bmatrix} -- \\ -- \\ -- \\ -- \\ -- \\ -- \\ -- \end{bmatrix} = Mx' + e_i \tag{2}$$

3)Detecting the error e_i. Multiply the information vector you just received by the Hamming matrix H:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}}_{H} . \underbrace{\begin{bmatrix} -- \\ -- \\ -- \\ -- \\ -- \\ -- \\ -- \end{bmatrix}}_{Mx'+e_i} (= H.Mx' + He_i = 0 + H.e_i) = \underbrace{\begin{bmatrix} -- \\ -- \\ -- \end{bmatrix}}_{H.e_i} \tag{3}$$

Notice that the columns of H are all different and the result He_i is one of these columns. Its number should tell you what i is and hence where the error is.

4) Decoding. Having found the position i of the error, you can go back to the original vector you received from your neighbour and decode his message - notice that $x' = (d'_1, d'_2, d'_3, d'_4)$ is part of $Mx' = (p'_1, p'_2, p'_3, d'_1, d'_2, d'_3, d'_4)$. Now lookup x' in the encoding table, so you can write down the symbol your neighbour chose: ____________ . Check with your neighbour whether you've decoded his message successfully.