Math 21b, Section 7, In-class exercise
Fourier series

Find the Fourier series of the function

$$f(x) = |x|$$

on $[-\pi, \pi]$.

Is f even, odd (or neither)?

Are any of the Fourier coefficients going to be 0 then?

Compute the Fourier series for $f(x) = |x|$:

$$a_n =< f(x), \cos(nx) >= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx =$$
$$\frac{1}{\pi}\int_{-\pi}^{\pi} |x|\cos(nx)dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\cos(nx)dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos(nx) =$$
$$= ...$$

$$b_n =< f(x), sin(nx) >= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sin(nx) = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|sin(nx)$$
$$= ...$$