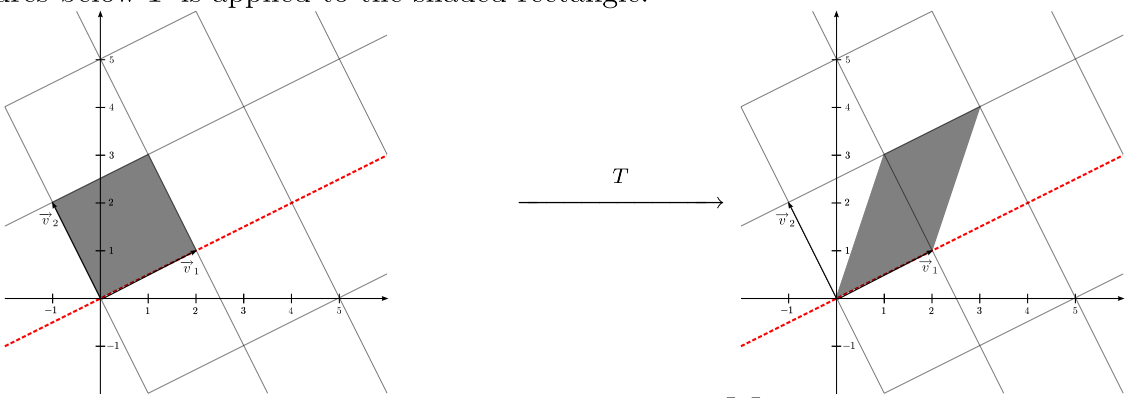
Math 21b, TTh 11:30 Section, Lecture 8, In Class Exercise
Coordinates, Bases, Change of bases

Consider a shear T along the dashed line L spanned by the vector $\vec{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, in the pictures below T is applied to the shaded rectangle.



We are interested in finding the image of the vector $\vec{x} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ under this transformation, i.e. find $T(\vec{x})$.

Part I. First, find the coordinates of $\vec{x}$ with respect to the basis $\mathcal{B} = \{\vec{v_1}, \vec{v_2}\}$, where $\vec{v_2} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ is conveniently a side of the shaded rectangle. In order to do that form the matrix $S = \begin{bmatrix} | & | \\ \vec{v_1} & \vec{v_2} \\ | & | \end{bmatrix}$ and find the coordinates of $\vec{x}$ in $\mathcal{B}$ by the formula $[\vec{x}]_{\mathcal{B}} = S^{-1}.\vec{x}$. (You might want to use the formula for inverse of a 2×2 matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.)

Part II. What is the image of $\vec{v_1}$ and $\vec{v_2}$ under T as a linear combination of $\vec{v_1}$ and $\vec{v_2}$, i.e. fill in the gaps:

$$T(\vec{v_1}) = \_\vec{v_1} + \_\vec{v_2},$$

$$T(\vec{v_2}) = \_\vec{v_1} + \_\vec{v_2}.$$

Now you can write the matrix of T in the basis $\mathcal{B}$:

$$B = \begin{bmatrix} | & | \\ [T(\vec{v_1})]_{\mathcal{B}} & [T(\vec{v_2})]_{\mathcal{B}} \\ | & | \end{bmatrix} = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}.$$

Part III. Finally, find $T(\vec{x})$. First find $[T(\vec{x})]_{\mathcal{B}} = B.[\vec{x}]_{\mathcal{B}}$. Then find $T(\vec{x}) = S.[T(\vec{x})]_{\mathcal{B}}$ in our usual basis.