Math 21b, TTh 11:30 Section, Lecture 6, In Class Exercise
Bases and Linear Independence

Let V be the subspace of $\mathbb{R}^3$, spanned by the vectors

$$\vec{v_1} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \; \vec{v_2} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \; \vec{v_3} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \; \vec{v_4} = \begin{bmatrix} 3 \\ -6 \\ -2 \end{bmatrix}. \tag{1}$$

Q1: Are $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ linearly independent? Hint: consider the linear system:

$$\underbrace{\begin{bmatrix} 2 & 0 & 2 & 3 \\ 0 & 2 & 2 & -6 \\ 2 & 1 & 3 & -2 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

Make a "guess" for the answer to Q1 before you solve the system: how many solutions could a system of 3 equations and 4 variables have and how many solutions would there be if the vectors were linearly independent?

Q2: Are $\vec{v_3}$ and $\vec{v_4}$ redundant? Hint: Is there a solution to the above system of the form $(a, b, c, 0)$ with $c \neq 0$ and another solution that looks like (a, b, c, d) with $d \neq 0$?

Q3: After answering Q2 find vectors which form a basis for V. Hint: Are $\vec{v_1}$ and $\vec{v_2}$ linearly independent and if so do they span V? How many are these basis vectors and what is $rank(A)$?

Q4: Is the vector $\vec{u} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ in V? hint: Can you express it as a linear combination of the basis vectors you found in Q3.