Math 19a, readings 23.2 and 24.1 review

April 22, 2008

1 Snowshoe Hare Populations: Squeezed from Below and Above; Impact of Food and Predation on the Snowshoe Hare cycle

This is an especially interesting article that tries to identify how many variables are defining the oscillatory variation in the hare population. A logical assumption is that hare population, predator population and hare food (e.g. grass) amount are related, but which relation is responsible for the oscillation (periodicity) in the hare population. In order to determine this, the authors performed an ecological experiment - they enclosed areas in the hare's natural habitat (southern Yukon) where they were able to control food supply and/or predator access. They observed that changing only one factor (either doubling food or decreasing predation about 2 times) leads to doubling or tripling of the hare population, however changing both factors simultaneously increased population by 11. This is not an additive effect, so there must be a three-fold interaction here (this conclusion is based on research the authors cite, which is not included in our book).

The principle used in this reasoning is that if you had a one parameter system (e.g. $\frac{dh}{dt} = f(h)$, where $h(t)$ is the hare population t time t), then $h(t + 1)$ is a function of $h(t)$ (i.e. if you plot $h(t+1)$ versus $h(t)$, i.e. the points $(h(t), h(t+1))$, you'll get a line, or in other words $h(t + 1)$ depends solely on $h(t)$) (this is not really obvious, but is similar to saying that $\frac{dh}{dt}$ depends only on the value of h at time t, i.e. is a function of h). If you had a three-way interaction, e.g. a system of the kind you might use in this article

$$\frac{dh}{dt} = f_1(h, g, p)$$
$$\frac{dp}{dt} = f_2(h, g, p)$$
$$\frac{dg}{dt} = f_3(h, g, p),$$

where p and g are predator population and food amount, then $h(t + 3)$ will be a function of $(h(t), h(t + 1), h(t + 2))$, or in other words knowing the values at time t of h, $\frac{dh}{dt}$, $\frac{d^2h}{dt^2}$ determines uniquely the value of $\frac{d^3h}{dt^3}$.

2 Disparate Rate of Molecular Evolution in Cospeciating Hosts and Parasites

This is an example of interaction between systems with different speeds. In other words, a fast system (lice) interacting with a slow one (gopher). The speed here refers to the production a new generations (or turnover rate of generations). The gopher generation is approximatelly three times longer (slower) than the lices. For that reason, the lice has a time to evolve to perfect adaptation to its host every time the gopher changes before there is any new change by the gopher. The system could be modeled perhaps using a competing species model, however instead of having population abundances as variables, we can consider allele frequencies. For example, if we consider a mutation in the gopher, which makes his skin harder (to

endure the lice better) and denote the percentage of gophers with such allele by g, then we can consider an adaptive mutation in the lice, which makes say their teeth sharper and longer to bite the hard skin of the gopher, denote the abundance of this allele by l. Then we can model g and l similarly to competing species (since the mutation in one species is bad for the other).

The authors however do not model this situation. Instead they try to show evidence to what they see by correlating the perfect adaptation of the lice to the genome of the gopher. A phylogenetic analysis is made to relate the adaptation of a protein that has a high rate of variability among gophers to lice, and it is found in almost all cases that the genome of the lice is highly correlated with the genome of its host. In fact the phylogenetic trees of hosts and parasites have similar topology (structure) which shows the correlation between mutations. This is one evidence that the fast paced evolution of a system with shorter generation time is able to adapt quickly to changes in the long paced one.