Math 19a, reading 21.1 review

April 15, 2008

1 Hantavirus Outbreak Leads to PCR

This article is about the sudden appearance of an absent so far type of virus in the US, the hantavirus. The virus causes the so called hantavirus pulmonary syndrome (HPS), which can lead to tachycardia, then to cardiovascular shock and eventually death. The virus is carried by rodents(they are vectors for the disease) and is passed to humans through air (aerosol from rodents' urine and feces). In 1993 26 cases in the South West have been reported and even though the virus has been expected to spread, it actually didn't and so the case of its appearance remains in the history so far.

The hantavirus outbreak case is important as the first case when scientists were able to identify and study the viral properties without having the virus itself. After they figured out that the virus in the American Southwest is similar to the Hantaan viruses in East Asia, they used DNA primers from the known Hantaan virus to start a Polymerase Chain Reaction (PCR) in tissues of infected patients and thus amplify and isolate the viral DNA present in the tissues. Even without the virus, having its genes is already a significant advantage to studying the virus and creating antiviral medications.

For our purposes this article is interesting in the sense that it motivates the study of the traveling wave solutions to the diffusion equations, which we did in chapter 21. In the case of a viral epidemic one of the first questions we'd ask is how fast does the virus spread and how soon will it reach us.

The model for this article is the one in chapter 21:

$$\frac{\partial}{\partial t} = \frac{\partial^2}{\partial x^2} u + ru(1-u). \tag{1}$$

Here $u(x,t)$ is the percentage of virus carrying mice at time t and position x (assuming they are moving along a line). The term $ru(1-u)$ is the reaction term - without the presence of diffusion, e.g. mice confined in a room, it shows the rate at which the virus spreads. This rate we chose as $ru(1-u)$, because it's our logistic model (we expect that eventually all mice get infected, so 1 is a stable equilibrium and 0 - unstable). The diffusion term $\frac{\partial^2}{\partial x^2} u$ indicates that mice are roaming around randomly, i.e. diffusing.

We are interested in a traveling wave solution, so $u(x,t) = f(x - ct)$. In this case the speed of the wave is c. In chapter 21 we established that in the case $c^2 > 2r$ there is always a solution f, such that

$$f(s) \to 1 \text{ as } s \to -\infty,$$

$$f(s) \to 0 \text{ as } s \to \infty,$$

$$0 < f(s) < 1 \text{ for all } s \text{ and } f \text{ is decreasing.}$$

This is a biologically relevant solution to our situation and its existence as long as $c > \sqrt{2r}$ shows that the speed of the wave can be very large, in other words our infection can be spreading really fast.