Reading Chapter 3, Fisheries

Greta Panova

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Give a sign up sheet. Try to learn names. Ask about the material, how do people feel about analyzing the behavior of differential equations, how about the logistic model. Will do some review of it now.

So how was the reading? What was most confusing? What is hard?

Let's see what is actually going on. Fish populations are dramatically decreasing. One factor is clearly the overfishing. That's intuitively clear, reproduction of fishes can't account for their mortality caused by catch. A very simple mathematical model of this would be to add a constant negative term to the exponential growth equation, e.g.

$$\frac{dp}{dt} = \alpha p - H$$

, where H is the amount of fish caught each year (if time is measured in years). So clearly if the population p falls bellow H/α, it will start decreasing and ultimately reach 0. 0, hm, is 0 an equilibrium here at all? No.

It's clear that overfishing is generally bad, but just stopping it is not always enough. This brings us to the concept of depensation. By definition: In population dynamics, depensation is the effect on a population (or stock) whereby, due to certain causes, a decrease in the breeding population (mature individuals) leads to reduced survival and production of eggs or offspring. The cause may be either:

predation levels rising per offspring (given the same level of overall predator pressure), or

the Allee effect, which is the reduced likelihood of finding a mate.

The general idea of the Allee effect is that for smaller populations, the reproduction and survival of individuals decrease. This effect usually saturates or disappears as populations get larger. The reasons for some species might be that reproductionfinding a mate in particularmay be increasingly difficult as the population density decreases. Other species may use strategies (such as schooling in fish) that are more effective for larger populations.

Depensation implies that below certain levels the population is likely to die out naturally, i.e. regardless of fishing. Why is there "good news" then if there is no depensation but still overfishing?

Let's consider the logistic model and try to apply it to our situation. The logistic model is

$$\frac{dp}{dt} = kp(1 - \frac{p}{N}).$$

If we account for the harvesting, then each year we catch a constant amount H, so we add a $-H$, to get $\frac{dp}{dt} = kp(1-p/N)-H$. Draw graph and show that if $p < p_1$ (the smaller root), then population dies out. So it's either overfishing killing it or there might be depensation. How to find out? - remove the harvesting term H, then the equation becomes $\frac{dp}{dt} = kp(1 - p/N)$, so 0 is an unstable equilibrium and if $p > 0$ and $p < N$ it will converge to N. Is there depensation then? No.

Let's consider a modified logistic model:

$$\frac{dp}{dt} = kp(1 - \frac{p}{N})(\frac{p}{M} - 1).$$

If we want to account for harvesting, we put again a $-H$ term there. Graph the model and analyze it. Assume $0 < M < N$. Show there are two stable equilibria - at $p = 0$ and at the carrying capacity $p = N$, and an unstable equilibrium at $p = M$, called "sparsity constant". So what happens if p drops below M? If $p < M$, then p will start decreasing until it reaches the next equilibrium, i.e. at 0, so population dies out. Is this depensation now? Yes.

Let's go back to the papers, reading 3.3. They consider the Beverton-Holt model without actually stating the model itself, so here it is:

The model is actually discrete. I.e. we don't measure the time continuously, but at separate steps. In this case we measure steps with the generations. We also divide the fish into two groups - spawners (mature fish producing spawn) and recruits (fish which developed from the spawn). We measure time in generations, so at the $n-$th generation, let R_n be the number of recruits and S_n - the abundance of spawners. So the recruits R_n will grow into spawners and the spawners will produce spawn and hence recruits.

So the n-th generation recruits will grow up (survive) with some probability, c, so $S_n = cR_n$.

On the other hand, the number of recruits which grow from the spawner's spawn is

$$R_{n+1} = \frac{\alpha S_n^\delta}{1 + \frac{S_n^\delta}{K}}.$$

What happens when $S_n \to \infty$? We have $R \to \alpha K$, which actually makes sense because due to overcrowding and limited resources the fish population can't grow unboundedly. This value αK is referred to as the "asymptotic recruitment". Explaining figure 1 - note that they are graphing their Beverton-Holt function in the limit values of S and R ($R_n \to R$ and $S_n \to S$), we have 50% asymptotic recruitment at the point $(S = 3, R = 11)$, so $\alpha K = 2 * 11 = 22$ and plugging in the equation $\frac{3\alpha K}{K+3} = 11$, so $K = 3$, $\alpha = 22/3$.

So we get a system

$$S_n = 2/5R_n, R_{n+1} = \frac{22S_n^\delta}{3 + S_n^\delta}.$$

We can find the equilibria - assuming $R_n \to R$ and $S_n \to S$, plug them in the system and solve - get exactly the intersection points of the sigmoidal curve and the 2/5 slope line -(0,0), . Which of these are actually stable - can analyze this via cobbwebbing:

Suppose $\delta = 1$, then only two solutions - (0,0) and (29/5, 29/2). To check for depensation - suppose $S_n < 29/5$, then $R_{n+1} = \frac{22S_n}{3+S_n} > 5/2S_n = R_n$, so $S_{n+1} = 2/5R_{n+1} > S_n$ and so both S_n and R_n will be increasing, i.e. going away from 0, no depensation.

Suppose now $\delta = 2$, then there are three equilibria: (0,0), (0.36, 0.89) and (8.4, 21). Then it's easy to see from the cobwebbing that (0,0) and (8.4, 21) are stable equilibria, while (0.36, 0.89) isn't and so there is depensation.

If time permits: connection with the continuous model. Let $\delta = 1$. Plugging $R_{n+1} = 1/cS_{n+1}$ in the second equation of the model we get

$$S_{n+1} = \frac{c\alpha S_n}{1 + S_n/K}$$

, so subtracting S_n from both sides we get $\Delta S_n = \frac{S_n(c\alpha - 1 - S_n/K)}{S_n/K+1} = (c\alpha - 1)\frac{S_n(1 - \frac{S_n}{K(c\alpha-1)})}{S_n/K+1}$, which is similar to the logistic equation and so we can see that for example $(c\alpha - 1)K$ is the carrying capacity. This also helps us analyze easily the situation and readily see that there is no depensation in this case and also that the equilibrium is $(c\alpha - 1)K = 29/5$.