Solution Supplement

4.7 – 12

b = length of base
h = height

$$V = b^2 h = 32{,}000$$

$$A = b^2 + 4hb \qquad \text{eliminate } h: \quad h = \frac{32{,}000}{b^2}$$

$$\Rightarrow \quad A = b^2 + \frac{4 \cdot 32{,}000}{b}$$

find critical pts: $A' = 2b - 4 \cdot 32{,}000/b^2$

$= 0$ when $b = \sqrt[3]{64{,}000} = 40$

since $A \to \infty$ as $b \to 0^+$ or $b \to \infty$, ~~this is the~~

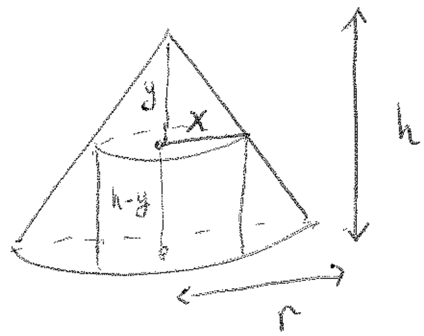
A must have an absolute min at some local min.

$b = 40$ is the only ~~local min~~ critical value

$\Rightarrow$ $b = 40$ is the abs. min

find h: $h = \dfrac{32{,}000}{40^2} = 20$

4.7 - 28



by similar triangles: $\frac{y}{x} = \frac{h}{r} \Rightarrow y = hx/r$

$$Vol = V = \pi x^2(h-y) = \pi h x^2 - \left(\frac{\pi h}{r}\right) x^3$$

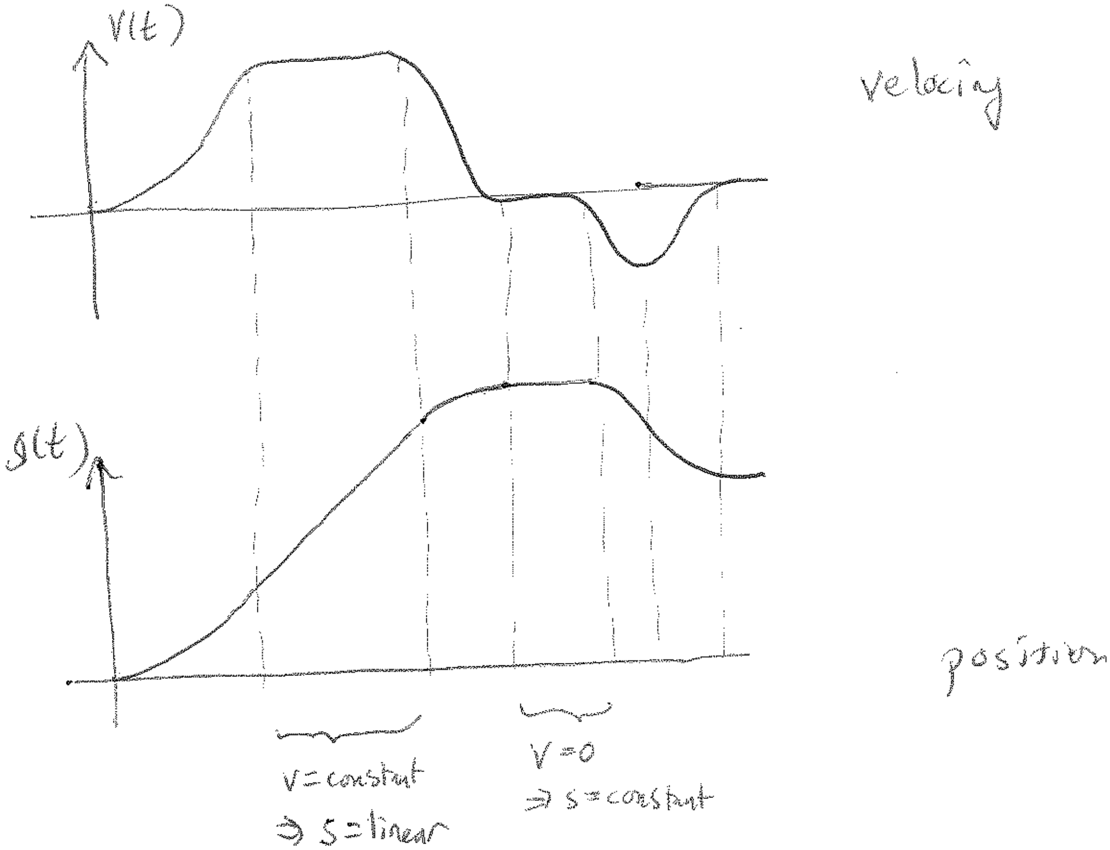
$$V' = 2\pi h\, x - \frac{3\pi h}{r} x^2$$

$$= 0 \text{ when } x = 0 \text{ or } x = \tfrac{2}{3} r$$

$V(0) = 0$, so maximum is at $x = \frac{2}{3} r$.

$$\boxed{V\left(\tfrac{2}{3} r\right) = \frac{4}{27} \pi r^2 h}$$

4.9 -46



$v > 0 \Rightarrow s$ increasing
$v < 0 \Rightarrow s$ decreasing

4.9-54

$a(t) = v'(t) = \cos t + \sin t$

$\Rightarrow v(t) = \sin t - \cos t + C$

$v(0) = 5 \Rightarrow C = 6$

So $v(t) = s'(t) = \sin t - \cos t + 6$

$\Rightarrow s(t) = -\cos t - \sin t + 6t + D$

$s(0) = 0 \Rightarrow D = 1$

So $s(t) = -\cos t - \sin t + 6t + 1$

5.2 - 48

$$\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$$

5.3 - 32

$$\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta = [\sec\theta]_0^{\pi/4} = \sec \pi/4 - \sec 0 = \sqrt{2} - 1$$

5.3 - 70

$$\int_0^1 10^x \, dx = \left[\frac{10^x}{\ln 10}\right]_0^1 = \frac{9}{\ln 10}$$

5.3 - 72

$$\int_0^1 \frac{4}{t^2+1}\,dt = 4\int_0^1 \frac{1}{1+t^2}\,dt = 4\left[\tan^{-1} t\right]_0^1 = \pi$$

5.3 - 74

$$\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 4u^{-3} + u^{-1}\,du = \left[-\frac{4}{2}u^{-2} + \ln|u|\right]_1^2$$

$$= \frac{3}{2} + \ln(2)$$