Solution Supplement

7.2 - 48

$$y' = \frac{xe^x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$$

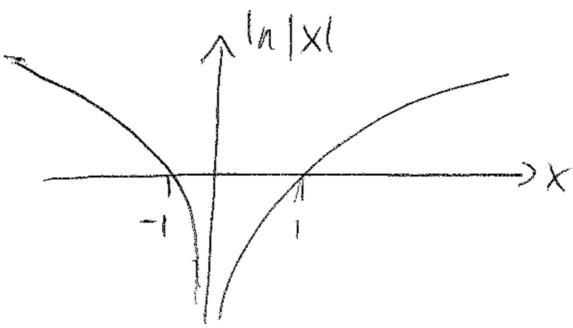
$\Rightarrow$ tangent: $y - e = 0(x-1)$

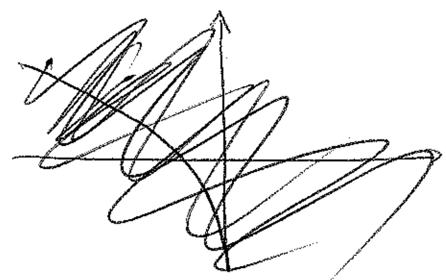
i.e. $y = e$

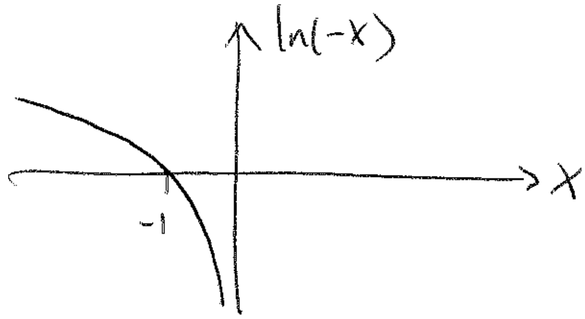
7.3 - 10

$$\frac{1}{2}\ln a + \frac{1}{2}\ln(b^2+c^2)$$

7.3 - 24







7.4 - 6

$$f(x) = \frac{\ln(xe^x)}{\ln(5)} \rightarrow f'(x) = \frac{1}{\ln(5)} \cdot \frac{1}{xe^x} \cdot (xe^x)'$$

$$= \frac{1}{\ln 5} \frac{(xe^x + e^x)}{xe^x}$$

7.4 – 46

$$\ln y = \cos x \ln x$$

$$\rightarrow \frac{y'}{y} = \cos x \cdot \frac{1}{x} + \ln x \cdot (-\sin x)$$

$$\rightarrow y' = \left(x^{\cos x}\right) \cdot \left(\frac{\cos x}{x} - \sin x \cdot \ln(x)\right)$$

7.5 – 2

a) $P(t) = 60e^{kt}$ t in hours

$P(\frac{1}{3}) = 120$ since $t = \frac{1}{3}$ hr $= 20$ min gives $P = 120$

$$\Rightarrow 60e^{K/3} = 120$$

$$\Rightarrow e^{K/3} = 2$$

$$\Rightarrow K/3 = \ln 2 \rightarrow K = \underline{\underline{3 \cdot \ln 2}}$$

b) $P(t) = 60e^{(3\cdot\ln 2)t} = 60 \cdot 8^t$

c) $P(8) = 60 \cdot 2^{24} \approx 1$ billion

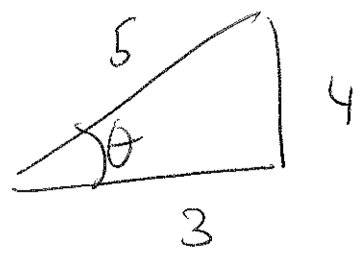
d) $P'(8) = KP(8) \approx 2.093$ billion cells/hr

e) $P(t) = 20{,}000$

$$\Rightarrow 60 \cdot 8^t = 20{,}000 \Rightarrow 8^t = 1000/3$$

$$\Rightarrow t\ln 8 = \ln\left(\frac{1000}{3}\right) \rightarrow t = \frac{\ln(1000/3)}{\ln 8} \approx 2.79$$

7.6 - 8



$\theta = \arccos 3/5$

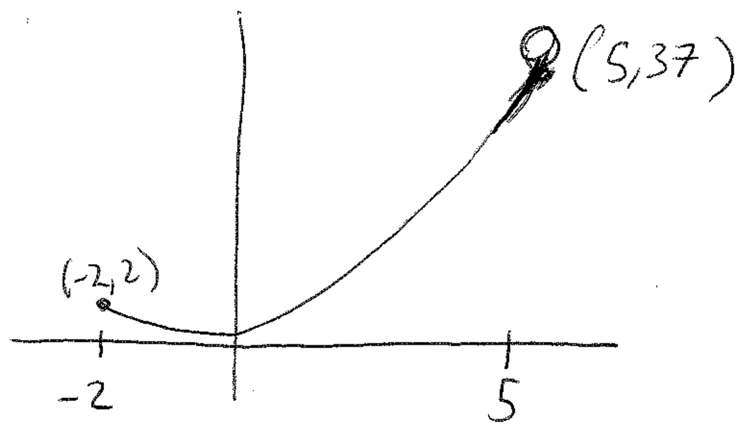
$\Rightarrow \csc\theta = 5/4$

7.6 - 28

$$F'(\theta) = \frac{1}{\sqrt{1-(\sqrt{\sin\theta})^2}} \frac{d}{d\theta}(\sin\theta)^{1/2}$$

$$= \frac{1}{\sqrt{1-\sin\theta}} \frac{1}{2}(\sin\theta)^{-1/2} \cdot \cos\theta$$

4.1 - 22



no local max, no abs max

absolute and local min at $x = -1$

4.1 - 50

$f'(x) = 3(x^2-1)^2 2x = 6x(x+1)^2(x-1)^2$

critical #'s : $x = -1, 0, 1$

x	$f(x)$	
-1	0	
0	-1	$\rightarrow$ min
1	0	
2	27	$\rightarrow$ max

abs max at $x=2$,
abs min at $x=0$

4.1 - 54 $\quad f(t) = \sqrt[3]{t}\,(8-t)$

$\Rightarrow f'(t) = \frac{8}{3}t^{-2/3} - \frac{4}{3}t^{1/3}$

$=0$ when $t=2$,
and $f'(t)$ does not exist for $t=0$ } critical #'s

t	$f(t)$	
0	0	both = abs min
2	$6\sqrt[3]{2}$	$\leftarrow$ abs max
8	0	both = abs min

4.2-17 $f(x) = 1 + 2x + x^3 + 4x^5$

By intermediate value thm, $f(c) = 0$ for some c between -1 and 0

if there are numbers a and b where $a < b$ and $f(a) = f(b) = 0$ then $f'(r) = 0$ for some r between a and b, by Rolle's thm.

But $f'(x) = 2 + 3x^2 + 20x^4 \geq 2$,

so no such r exists.

4.2-34 if $f(a) = a$, $f(b) = b$ and $a \neq b$,

then by MVT, there is a number c such that $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{b-a}{b-a} = 1$,

a contradiction.

4.3 - 8

a) incr on $(2,4) \cup (6,9)$

b) loc max : $x=4$

loc min : $x=2, x=6$

c) concave up : $(1,3) \cup (5,7) \cup (8,9)$

concave down : $(0,1) \cup (3,5) \cup (7,8)$

d) $x = 1, 3, 5, 7, 8$

4.3 - 38)

$$G'(x) = 1 - \frac{2}{\sqrt{x}}$$

$G'(x) > 0$ when $x > 4$ (increasing)

$G'(x) < 0$ when $0 < x < 4$ (decreasing)

$G'(x) = 0$ when $x = 4$ (local min)

no local max

$$G''(x) = x^{-3/2} > 0$$

so always concave up, no inflection pts

4.4 -18

$$\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x} \div x^3}{(x^3+1) \div x^3}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{9x^6 - x} \div (-\sqrt{x^6})}{(1 + \frac{1}{x^3})}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{9 - 1/x^5}}{1 + \frac{1}{x^3}}$$

7.2 - 24

$$\lim_{x \to -\infty} a^x = 0 \quad \text{for } a > 1$$

7.3 - 50

$$= \lim_{x \to \infty} \ln\left(\frac{2+x}{1+x}\right) = \lim_{x \to \infty} \ln\left(\frac{2/x+1}{1/x+1}\right)$$

$$= \ln 1 = 0$$