HW3 - solution supplement

3.6 - 12)

$$\frac{d}{dx}(1+x) = \frac{d}{dx}\sin(xy^2)$$

$$1 = \cos(xy^2)\frac{d}{dx}(xy^2)$$

$$1 = \cos(xy^2)(y^2 + 2xyy')$$

$$1 - \cos(xy^2)y^2 = 2xy\cos(xy^2)y'$$

$$\Rightarrow y' = \frac{1-\cos(xy^2)y^2}{2xy\cos(xy^2)}$$

3.6 - 20) $\frac{d}{dx}(\sin x + \cos y) = \frac{d}{dx}(\sin x\cos y)$

$$\cos x - \sin y \cdot y' = \cos x\cos y - \sin x\sin y \cdot y'$$

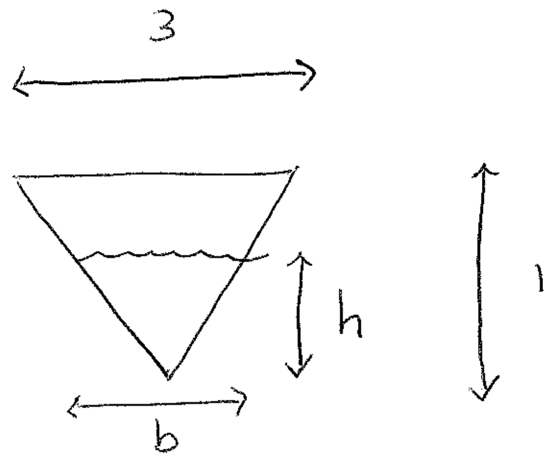
$$\cos x - \cos x\cos y = \sin y \cdot y' - \sin x\sin y \cdot y'$$

$$\cos x(1-\cos y) = y' \cdot \sin y(1-\sin x)$$

$$\Rightarrow y' = \frac{\cos x(1-\cos y)}{\sin y(1-\sin x)}$$

3.8 — 24)

diagram:



by similar triangles: $\frac{b}{h} = \frac{3}{1}$

$\rightarrow b = 3h$

Volume formula: $V = (\frac{1}{2} bh)(10) = 5bh$

substitute for b: $V = 15h^2$

We are Given $\frac{dV}{dt}$, and we want $\frac{dh}{dt}$. Take $\frac{d}{dt}$ of both sides:

$$\frac{dV}{dt} = 30h \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{1}{30h} \cdot \frac{dV}{dt}$$

plug in numbers:

$$\frac{dh}{dt} = \frac{1}{30 \cdot \frac{1}{2}} 12 = \frac{4}{5} \text{ ft/min}$$

3.9, −34)

Volume of hemisphere $= \frac{2}{3}\pi r^3 = V(r)$

Volume of hemispherical shell $= V(r+\Delta r) - V(r) = \Delta V$

use approx $\Delta V \approx dV$ and compute dV

$$dV = \frac{dV}{dr} \cdot dr = (2\pi r^2)\,dr$$

$$= (2\pi)\cdot\left(\frac{50}{2}\right)^2 \cdot (0.0005)$$

$$= \frac{5\pi}{8} \approx 2\ m^3$$

diameter $= 50\ m \Rightarrow r = \frac{50}{2}\ m$

thickness $= 0.05\ cm \Rightarrow dr = 0.0005\ m$

7.1 − 20)

a) passes horizontal line test

b) domain of f $= [-3, 3]$, range of f $= [-1, 3]$

$\Rightarrow$ domain of f^{-1} $= [-1, 3]$, range of f^{-1} $= [-3, 3]$

c) since $f(0) = 2$, $f^{-1}(2) = 0$

d) $f(-1.7) \approx 0$, so $f^{-1}(0) \approx -1.7$

7.1 − 34) a) suppose $f(x_1) = f(x_2)$

to show: $x_1 = x_2$

Proof: $f(x_1) = f(x_2)$

$\to \sqrt{x_1 - 2} = \sqrt{x_2 - 2}$

$\to x_1 - 2 = x_2 - 2$

$\to x_1 = x_2$

c) $y = \sqrt{x-2}$

$\Rightarrow y^2 = x - 2$

$\Rightarrow x = y^2 + 2$

$\Rightarrow f^{-1}(x) = x^2 + 2$

domain of $f^{-1}(x)$

$= $ range of $f(x)$

$= [0, \infty)$

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range of $f^{-1}(x)$

$=$ domain of $f(x)$

$= [2, \infty)$

b) and d): find $(f^{-1})'(2)$

using theorem 7: $(f^{-1})'(2) = \dfrac{1}{f'(f^{-1}(2))}$

$$f'(x) = \frac{1}{2\sqrt{x-2}}$$

$$f(6) = 2 \Rightarrow f^{-1}(2) = 6$$

$$\therefore (f^{-1})'(2) = \frac{\cancel{\quad}}{2\sqrt{6-2}} = 4$$

using formula for $f^{-1}(x)$:

$$f^{-1}(x) = x^2 + 2$$

$$\Rightarrow (f^{-1})'(x) = 2x$$

$$\Rightarrow (f^{-1})'(2) = 4$$