HW sol. 1

1.1

2) a) $f(-4) = -2$

$g(3) = 4$

b) $x = -2$ and $x = 2$

c) $x = -3$, $x = 4$

d) $[0,4]$

e) f has domain $[-4,4]$

f has range $[-2,3]$

f) g has domain $[-4,3]$

g has range $[\frac{1}{2}, 4]$

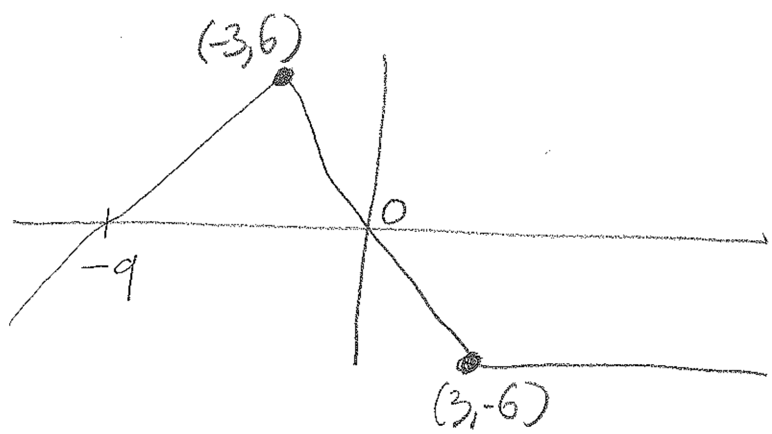
28) domain is all x except when $\underbrace{x^2+3x+2=0}_{\iff x=-2, \text{ or } -1}$

domain $= (-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$

30) g is defined when $u \geq 0$ and $4-u \geq 0 \iff u \leq 4$

domain $= [0,4]$

44)



domain $= \mathbb{R}$

1.1 68) f is odd:

$$f(-x) = -x \cdot |-x| = -x \cdot |x| = -f(x)$$

1.2 4) f $y = 3^x$ g $\sqrt[3]{x}$

F $y = x^3$ G $y = 3x$

1.3 6) $y = 2f(x-2)$ (shift right by 2 & stretch vertically by a factor of 2)

$$= 2\sqrt{3(x-2)-(x-2)^2}$$

$$= 2\sqrt{-x^2+7x-10}$$

42) $f(x) = \sin(x)$, $g(x) = \sqrt{x}$

44) $f(x) = \sqrt[3]{x}$, $g(x) = \frac{x}{1+x}$

46) $f(x) = \frac{x}{1+x}$, $g(t) = \tan(t)$

50) a) 5
b) 2
c) 4
d) 3
e) 1
f) 4

2.2 26) $\lim_{x \to -3^-} \frac{x+2}{x+3} = \infty$

Since: - numerator is < 0 near $x = -3$
- denom. approaches 0 from negative values

28) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty$ Since:

- x^2 approaches zero from positive values
- $x+2$ is >0 near $x=0$
- $x-1$ is <0 near $x=0$

32) $\lim_{x \to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2}$

$= \lim_{x \to 2^-} \frac{x}{x-2} = -\infty$

Since $x > 0$ near $x = 2$
and $x-2$ approaches zero from negative values

2.3 48) i) 1
ii) 1
iii) 3
iv) -2
v) -1
vi) does not exist

2.4 19) $|f(x)-L| = \left|\frac{x}{5}-\frac{3}{5}\right| = \frac{|x-3|}{5}$

this is $< 5\varepsilon \cdot \frac{1}{5} = \varepsilon$ if $|x-3|<5\varepsilon$

so pick $\delta = 5\varepsilon$

20) $|f(x)-L| = \left|\frac{x}{4}+3-\frac{9}{2}\right| = \frac{|x-6|}{4}$

this is $< 4\varepsilon \cdot \frac{1}{4} = \varepsilon$ if $|x-6|<4\varepsilon$

so pick $\delta = 4\varepsilon$

23) $|f(x)-L| = |x-a|$

pick $\delta=\varepsilon$

24) $|f(x)-L| = 0$, and 0 is always $<\varepsilon$.

so pick $\delta=1$ (or any other number)

25) $|f(x)-L| = x^2$, this is $< (\sqrt{\varepsilon})^2 = \varepsilon$

if $|x|<\sqrt{\varepsilon}$, so pick $\delta=\sqrt{\varepsilon}$.

39) given a number L, show $\lim_{x\to 0} f(x) \neq L$:

if $|1-L| \geq 1/2$ then given any $\delta>0$ pick x to be any irrational number s.t. $0<|x|<\delta$.
Then $|f(x)-L| = |1-L| \geq 1/2$.

if $|L| \geq 1/2$ then given any $\delta>0$ pick x to be any rational number s.t. $0<|x|<\delta$.
Then $|f(x)-L| = |0-L| \geq 1/2$.

This proves the limit is not L (using $\varepsilon = 1/2$).

[one of $|1-L| \geq 1/2$ and $|L| \geq 1/2$ will be true for any number L]

42) $f(x) = \dfrac{1}{(x+3)^4} = \dfrac{1}{|x+3|^4}$

this is $> M$ if $|x+3| < M^{-1/4}$.

so pick $\delta = M^{-1/4}$.

44) a) given $M>0$, pick $\delta_1 > 0$ s.t.

$0<|x-a|<\delta_1 \implies |g(x)-c|<1$. $(*)$

and pick $\delta_2 > 0$ s.t.
$0<|x-a|<\delta_2 \implies f(x) > M-c+1$ $(**)$

For $\delta = \min(\delta_1, \delta_2)$ one has

$$f(x)+g(x) \underset{\substack{\uparrow \\ \text{by } (*)}}{>} f(x) + (c-1) \underset{\substack{\uparrow \\ \text{by } (**)}}{>} (M-c+1) + (c-1) = M$$

44 b) Given $M>0$ pick $\delta_1 > 0$ s.t.

$$0 < |x-a| < \delta_1 \Rightarrow |g(x) - c| < c/2 \qquad (*)$$

and pick $\delta_2 > 0$ s.t.

$$0 < |x-a| < \delta_2 \Rightarrow f(x) > 2M/c \qquad (**)$$

then set $\delta = \min(\delta_1, \delta_2)$. For $0<|x-a|<\delta$, one has

$$f(x)g(x) \underset{\text{by } (*)}{>} f(x) \cdot c/2 \underset{\text{by } (**)}{>} \frac{2M}{c} \cdot c/2 = M$$

to see this: $|g(x) - c| < c/2$

$$\Leftrightarrow -c/2 < g(x) - c < c/2$$

$$\Leftrightarrow c - c/2 < g(x) < 3c/2$$

$$\Rightarrow g(x) > c/2$$

the choice of "$< c/2$" ensures that "$c - c/2$" is positive

31) $|f(x)-L| = |x^2-4| = |x-2||x+2|$

If $|x+2|<1$, then $-1 < x+2 < 1$

$\Rightarrow \quad -5 < x+2 < -3$

$\Rightarrow \quad |x+2| < 5$

If $|x+2| < \varepsilon/5$ and $|x+2| < 1$ then

$|f(x)-L| < \varepsilon/5 \cdot 5 = \varepsilon.$

pick $\delta = \min(\varepsilon/5, 1)$

36) $|f(x)-L| = \left|\frac{1}{x} - \frac{1}{2}\right| = \dfrac{|x-2|}{2|x|}$

if $|x-2|<1$, then $-1 < x-2 < 1$

$\Rightarrow \quad 1 < x < 3$

$\Rightarrow \quad |x| > 1$

$\Rightarrow \quad \dfrac{1}{2|x|} < \dfrac{1}{2}$

if $|x-2| < 2\varepsilon$ and $|x-2| < 1$ then

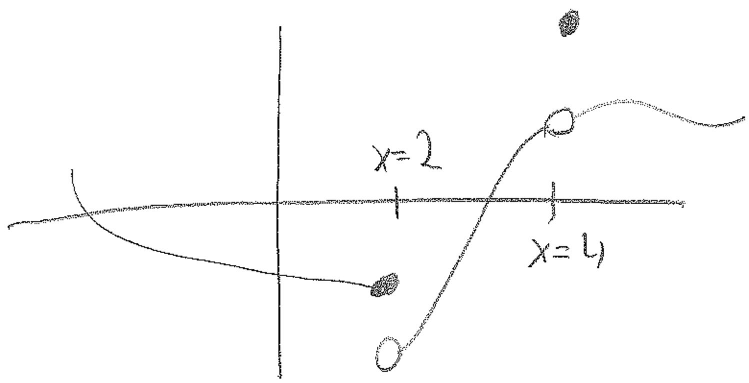
$|f(x)-L| < (2\varepsilon)\cdot\frac{1}{2} = \varepsilon.$

so pick $\delta = \min(1, 2\varepsilon)$.

2.5

4) $[-4,-2)$, $(-2,2)$, $[2,4)$, $(4,6)$ and $(6,8)$

6)



18) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{x^2-x}{x-1} = \lim\limits_{x\to 1} x = 1 \underbrace{\neq}_{\text{so not continuous}} f(1) = 0$