MATH 361 — HOMEWORK 11.

due on Friday, December 04.

Textbook: *“Elementary Classical Analysis”* , second edition
by J. E. Marsden and M. J. Hoffman

Topics:

- **Chapter 6:** Differentiable Mappings
  - 6.1 Definition of the Derivative
  - 6.2 Matrix Representation
  - 6.3 Continuity of Differentiable Mappings; Differentiable Paths
  - 6.4 Conditions for Differentiability
  - 6.5 The Chain Rule
  - 6.6 Product Rules and Gradients
  - 6.7 The Mean Value Theorem
  - 6.8 Taylor's Theorem and Higher Derivatives
- **Differentiability of Multilinear Maps and Inverses. Operations on Functions (The Lectures)**
- **Higher Derivatives (The Lectures)**

Eleventh Homework Assignment.

Reading:

- Read Section 6.8. Read the slides (or/and watch the videos).

Exercises: *(In what follows E and F are Banach Spaces).*

Problem 1. Compute the second derivative of a continuous linear map $T \in L(E, F)$.

Problem 2. Prove that every continuous $k - linear$ map $\phi \in L^{(k)}(E_1 \times \cdots \times E_k; F)$ is twice differentiable and compute its second derivative. What about higher derivatives?

Problem 3. Prove that the inverse map $\mathrm{Inv} : GL(E) \to L(E)$ is twice differentiable and compute its second derivative. What about higher derivatives?

Problem 4. Suppose that $U \subset E$ is an open set and that $f : U \to F$ is $n-$times differentiable. Suppose $x \in U$ and $h \in E$ and consider the function

$$\varphi(t) = f(x + th)$$

defined for all values of $t \in \mathbb{R}$ for which $x + th \in U$. Prove that the domain of definition of φ is an open set, that φ is $n-$times differentiable and that

$$\frac{d^n\varphi}{dt^n}(t) = D^n f(x+th)(h, h, \ldots, h).$$

Problem 5. (Page 388 - 31) Let (K, d) be a compact metric space and consider the Banach space $(C(K, \mathbb{R}), \| \|_\infty)$. Define for $x_0 \in K$, $\delta_{x_0} : C(K, \mathbb{R}) \to \mathbb{R}$; $f \mapsto f(x_0)$. Prove that δ_{x_0} is differentiable.

***Problems*:**

- Page 338: problems 5
- Page 362: problems: $1, 5$
- Page 383: problems:32