MATH 241 — HOMEWORK 1.

due on Friday, September 11.

Textbook: *"Applied Partial Differential Equations with Fourier Series and Boundary Value Problems"* , fifth edition
by Richard Haberman

Topics:

- Review of Prerequisite Concepts
  - Complex Numbers
  - Ordinary Differential Equations
  - The Divergence Theorem

First Homework Assignment.

Reading:

- Read your notes.
- Read your Math 114 and Math 240 sources for the Divergence Theorem and for the solutions of ODE's of the form

$$\frac{dy}{dx} + p(x)y = q(x).$$

Exercises:

Problem 1. Give the formula for $\cos 7\theta$ in terms of $\sin\theta$ and $\cos\theta$, by using Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta,$$

and the binomial formula.

Problem 2. Prove that if $z \in \mathbb{C}$ is any complex number different from 1, then

$$1 + z + z^2 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}.$$

Problem 3. Use Problem 2. and Euler's formula to give a formula for

$$\sin\theta + \sin 2\theta + \cdots + \sin n\theta$$

in terms of $\sin\theta, \cos\theta, \sin(n+1)\theta$ and $\cos(n+1)\theta$.

Problem 4. Find a solution $u = u(x, y)$ of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

subject to the condition $u(1,\theta) = \cos(3\theta) - 2\sin(4\theta)$ (written in polar coordinates (r,θ)).

Write your solution both as a function of x and y, and as a function of r and θ (polar coordinates).

Problem 5. Find the real and the imaginary parts of the function

$$f(z) = \frac{z+1}{2z+5},$$

where $z = x + iy$.

Problem 6. Find the general solution of the O.D.E.

$$\frac{d^2y}{dx^2} - (1+x^2)y = 0,$$

by noting first that it can be written as

$$(\frac{d}{dx} + x)(\frac{d}{dx} - x)y = 0.$$

(You might not be able to compute some integrals explicitly.)