Homwork 6

Problem 1. Let f be the function

$$f(x, y) = \ln(x + y)$$

for every $(x, y) \in \mathbb{R}^2$ and $x + y > 0$. A unit vector in $\mathbb{R}^2$ is a vector of length 1. What is the maximum value of the directional derivative $D_{\vec{u}}(f)$ of f at the point $(x, y) = (2, -1)$ as $\vec{u}$ ranges over all unit vectors in $\mathbb{R}^2$.

Problem 2. Let $T(x, y) = x^2 + y^2 - x - y$ be the temperature of at the point (x, y) in the plane. A lizard sitting at the point $(1, 3)$ wants to increase his surrounding temperature as quickly as possible. In which direction should he move?

Problem 3. Let

$$f(x, y, z) = \ln(x^2 + y^2) - z^3.$$

Using the linearization of f at $(-1, 1, 1)$ estimate the value of $f(-0.9, 1.2, 1.1)$. (Your final answer can contain $\ln 2$.)

Problem 4. Find the local minimum of the following function

$$f(x, y) = x^3 - 3xy + y^2$$

Problem 5. Find all the critical points of the function $h(x, y) = 2x \sin(y) + y^2 - x^2$ and determine which is a local maximum, which is a local minimum and which is saddle point.