Homework 5

Problem 1. Which of the following limits exist?

a.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 - y^2}$$

b.

$$\lim_{(x,y)\to(0,0)} \frac{x - y}{x^2 + y^2}$$

c.

$$\lim_{(x,y)\to(0,0)} \frac{x + y}{\sqrt{x^2 + y^2}}$$

Problem 2. Does the following limit exist

$$\lim_{(x,y)\to(0,0)} \frac{(x^2 + y^2)\sin(x^2 + y^2)}{x^4 + y^4}?$$

If yes, find the limit.

Problem 3. The function $z = f(x, y)$ is given implicitly by the equation $z^3 + z = x^2 + y^2$. Note that when $x = 1$ and $y = 1$, $z = 1$ as well. Compute $\frac{\partial f}{\partial x}(1.1)$

Problem 4. Consider the surface $z = x^2 + x + 2y^2$. At what point (x_0, y_0, z_0) is the tangent plane parallel to the plane $x + 4y + z = 0$? What is the z coordinate of that point ?

Problem 5. Find the equation of the tangent plane to the surface

$$4x^4 + 2y^4 + z^4 = 22$$

at the point $(1, 1, 2)$.