Homework 11

Problem 1. Compute the line integral

$$\int_C \vec{F} \cdot d\vec{r}$$

of the vector field

$$\vec{F} = \langle 2xy^2 + 3xz^2, 2x^2y + 2y, 3x^2z - 2z \rangle$$

on the curve C given by

$$\vec{r}(t) = \langle \cos(2t) + 5\sin(5t), 6\sin(t) + 4\sin(5t), \cos(2t) + \cos(5t) \rangle$$

for $0 \leq t \leq \pi$.

Problem 2. Evaluate the integral

$$\int_C (y + \sin(e^{x^2}))dx - 2xdy,$$

where C is the circle $x^2 + y^2 = 1$ traveled counter-clockwise.

Problem 3. Let S be the portion of the surface $z = xy$ lying inside the cylinder $x^2 + y^2 \leq 1$. Compute the surface area of S.

Problem 4. Find the potential function of the following gradient fields:

$$\vec{F_1} = (-y\sin(x) + \sin(z), \cos(x) + z, x\cos(z) + y + 2z),$$

$$\vec{F_2} = (yze^{xyz} + 2x, xze^{xyz} + 2y, xye^{xyz} + 2z).$$

Problem 5. Compute the following line integrals using Greens theorem (C is assumed to be oriented counterclockwise):

- $\int_C x^2ydx + xy^3dy$ where C is the square with vertices (0, 0), (0, 1), (1, 0), (1, 1).
- $\int_C (x + 2y)dx + (x - 2y)dy$ where C is the curve determined by the arc of parabola $y = x^2$ from (0, 0) to (1, 1) and the line segment joining the same two points.
- $\int_C x^2dx + y^2dy$ where C is the curve determined by $x^6 + y^6 = 1$.