Math 114, Spring 2017

Practice Problems for Midterm 2

1. Compute the following limits or show that they do not exist:

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2\sin y}{x^2+y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^5}{x^4+y^4}$

   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^3-y^3}$

   (e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{x^3-y^3}$

2. Find all the critical points of the function $f(x,y) = 3x^2y+y^3-3x^2-3y^2$ and determine whether they are local minima, local maxima, or saddle points.

3. Find the absolute minimum and maximum of $f(x,y) = xy + x^2 - y^2$ on the square $0 \le x \le 2$, $0 \le y \le 2$.

4. Compute the following multiple integrals:

   (a) $\displaystyle\int_0^1 \int_0^\pi x\cos(xy)\,\mathrm{d}x\,\mathrm{d}y$

   (b) $\displaystyle\int_0^3 \int_{x^2}^9 \frac{x}{\sqrt{3x^2+y}}\,\mathrm{d}y\,\mathrm{d}x$

   (c) $\displaystyle\int_0^\pi \int_x^\pi \frac{\cos y}{y}\mathrm{d}y\,\mathrm{d}x$

   (d) Find the area of the region lying inside the cardiod $r = 2+\cos\theta$ and outside of the circle $r = 1$.

   (e) Find the area of the region lying inside the circle $x^2+y^2 = 4$ and outside of the lemniscate given by $r^2 = \cos 2\theta$.

5. Consider the tangent plane of the surface $x^2 + y^2z^2 = 8$ at the point $(2,-1,2)$. At which point does it intersect the z-axis?

6. Use a linear approximation at $(1,1)$ to approximate the value of $z(x,y) = xe^{y-x}$ at the point $(1.1, 0.9)$.

7. Consider a function $f(x, y, z)$ of the quantities $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, which in turn are itself functions of t, that is, $u = u(t)$, $v = v(t)$. Write an expression for $\frac{\partial f}{\partial t}$ in terms of partial derivatives of f with respect to x, y, z, and $u'(t)$, $v'(t)$.

8. Donald Trump has requested that a new hangar (in the shape of a rectangular parallelepiped) is built at Mar-a-Lago to accommodate the private jets of his close friends. It will be built using material that costs \$31k per square foot for the roof, \$27k per square foot for the two sides and back, and \$55k per square foot for the fancy front. If the hangar is to have an internal volume of 16,000,000 cubic feet, what are the dimensions that will minimize the total construction cost?

9. The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of 5,000 square yards, and fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

10. A consumer has \$600 to spend on two commodities, the first costs \$20 per unit and the second \$30 per unit. Suppose the utility derived by the consumer from x units of the first commodity and y units of the second commodity is given by the so-called *Cobb-Doublas utility function* $U(x, y) = 10x^{0.6}y^{0.4}$. How many units of each commodity should the consumer buy to maximize utility given his budgetary constraint?

11. Compute the volume between the graph $z = x^2 + y^2$ and the xy-plane over the region given by $x^2 + y^2 \leq 4$ and $x \leq 0$, $y \leq 0$.

12. Let $f(x, y) = \ln(x^2 + y^2)$.

    (a) Compute its directional derivative at the point $(1, 1)$ in the direction of $\vec{i} - \vec{j}$.

    (b) Find the largest value of its directional derivatives $\frac{\partial f}{\partial \vec{v}}(1, 1)$ where $\|\vec{v}\| = 1$.

13. Find $\partial z/\partial r$ when $r = \pi$ and $s = 0$ if $z = \sin(2x - y)$ and $x = r + \sin s$ and $y = rs$.