1 Math 141. Spring'00 (Professor Kanovich), Mid-term 1

Name (print): ______________________________

Signature: ______________________________

Use separate sheets for your solutions of the problems.
Give detailed explanations, as if your solution were going to be published.
Staple the pages together. Write neatly and organize your pages so that we can read them.

Problem 1 (10 points)

Find the area of the surface of revolution obtained by rotating $y = \sqrt{4x - x^2 - 3}$

(a) around the $x-$axis,

(b) around the $y-$axis.

Problem 2 (10 points)

Find the length of the curve: $x = \sqrt{1 - y^2}$, $0 \leq y \leq 1$.

Problem 3 (10 points)

Describe the curve given in Cartesian coordinates:

$$x^2 + 2y^2 = 1,$$

by an equation in polar coordinates (r, θ). Check your polar equation for $\theta = 0$, $\theta = \frac{\pi}{4}$, and $\theta = \frac{\pi}{2}$.

Problem 4 (10 points)

Consider the following power series:

$$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + \ldots \tag{1}$$

(a) Find the radius of convergence R.

(b) Does the series converge or diverge for $x = -R$, and likewise for $x = R$?

(c) Find a simple expression for the sum of the series when $|x| < R$.

Problem 5 (13 points)

Consider the following power series:

$$1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + 7x^6 - 8x^7 + \ldots \tag{2}$$

(a) Find the radius of convergence R.

(b) Does the series converge or diverge for $x = -R$, and likewise for $x = R$?

(c) Find a simple expression for the sum of the series when $|x| < R$.

Problem 6 (11 points)

(i) Find the Maclaurin series $\sum_{n=0}^{\infty} a_n x^n$ of the function: $\sin^2 x$.

(ii) Find the Maclaurin series $\sum_{n=0}^{\infty} b_n x^n$ of the function: $\cos^2 x$.

(iii) Check that: $\sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} b_n x^n = 1$.

Problem 7 (11 points)

How many terms of the Maclaurin series of $\sin^2 x$ must you use in order to calculate $\sin^2 \frac{\pi}{4}$ with an error less than 0.01 ?

Problem 8 (10 points)

Find the Maclaurin series of the following functions: $(1+x)^3$, $(1+x)^4$, $(1+x)^5$, and $\sqrt{1+x}$.

Problem 9 (15 points)

(a) Is it true or false[1] that the following sequence $x_1, x_2, \ldots, x_n, \ldots$ is increasing:

$$x_n := 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n+1). \tag{3}$$

(b) Is it true or false[1] that for any $n \geq 2$:

$$\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n. \tag{4}$$

(c) Prove the existence of the following limit:

$$\lim_{n\to\infty} (1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n) \tag{5}$$

[1]Explain your answer