# AN INTRODUCTION TO FINANCIAL MATHEMATICS

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ABSTRACT.

## 1. Lecture 12: Pricing and hedging of derivatives in continuous time models

We will discuss here pricing and hedging of European, American and Israeli contingent claims in market models where the stock evolution is described by the geometric Brownian motion. This material can be found in [2], [3], [4] and [1].

We consider the Black-Scholes financial market where the price of a bond at time $t$ is given by $B_t = B_0 e^{rt}$ with $r \geq 0$ being the interest rate and the price of a stock at time $t$ is given by the geometric Brownian motion $S_t = S_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma W_t)$ where $W_t$ is the standard Brownian motion starting at 0. The value of a self-financing portfolio corresponding to a trading strategy $\pi = (\beta_t, \gamma_t)$ is given by

$$X_t^\pi = x + \int_0^t \beta_u dB_u + \int_0^t \gamma_u dS_u.$$

Similarly to the discrete time case the fair price of derivatives is defined is based on the notion of hedging and the definitions are the same. Thus, the fair prices of derivatives are given in the continuous time case by the formulas similar to the discrete time case as stated in the following theorem.

1.1. **Theorem.** *Denote by $E^*$ the expectation with respect to the unique martingale measure $P^*$ in the Black-Scholes market and let $T$ be the horizon. Then the fair price $V$ of a contingent claim with a payoff process $R$ is given by*

*(i) in the European case:*

$$V = B_0 E^*(\frac{R_T}{B_T});$$

*(ii) in the American case:*

$$V = \sup_{0 \leq \tau \leq T} B_0 E^*(\frac{R_\tau}{B_\tau}),$$

*where the supremum is taken over the stopping times;*

*(iii) in the Israeli (game) case*

$$V = \inf_{0 \leq \sigma \leq T} \sup_{0 \leq \tau \leq T} B_0 E^*(\frac{R(\sigma,\tau)}{B_{\sigma\wedge\tau}}),$$

*where both the infimum and the supremum are taken over the stopping times.*

*Date*: November 7, 2016.

## References

[1] Yu. Kifer, *Game options*, Finance Stoch. 4(2000), 443–463.

[2] D. Lamberton and B. Lapeyre, *Introduction to Stochastic Calculus Applied to Finance*, Chapman& Hall/CRC, London, 2008.

[3] S. Shreve, *Stochasic Calculus and Finance*, vol. 2, Springer, 2004.

[4] R.J. Williams, *Introduction to the Mathematics of Finance*, AMS, Providence R.I., 2006.