

AN INTRODUCTION TO FINANCIAL MATHEMATICS

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Consider the trinomial market where $B_n \equiv 1$ (interest rate is zero) and the price of the stock at time n is given by the formula

$$S_n = \prod_{k=1}^n (1 + \rho_k), \quad S_0 = 1$$

where ρ_1, ρ_2, \dots are i.i.d. random variables such that ρ_i takes on values $-1/2, 1/2$ and 1 , each with probability $1/3$. The market is active only for two stages (days) i.e. the horizon is $N = 2$. We consider two payoffs: a put option payoff $f_n = (2 - S_n)^+$ and a call option payoff $f_n = (S_n - 2)^+$. Find

- 1) All martingale measures (the market is incomplete!);
- 2) The superhedging prices of the corresponding put and call European options;
- 3) The superhedging prices of the corresponding put and call American options;
- 4) Self-financing hedging portfolio strategies with the initial capital equal the corresponding superhedging prices for the European options from 2);
- 5) Self-financing hedging portfolio strategies with the initial capital equal the corresponding superhedging prices for the American options from 3);
- 6*) (bonus/honors question) The superhedging prices of the corresponding put and call game options where the payoff is as above and the penalty is 1 (i.e. if the seller cancels before the buyer exercises then the seller adds to the above payoff the penalty equal 1);

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