AN INTRODUCTION TO FINANCIAL MATHEMATICS

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1. Assignment No.3, 10/17/2016, Deadline November 7, 2016

1) Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\rho_1, \rho_2, ...$ be i.i.d. random variables such that $\rho_i > -1$ with probability one and there is a number $a > 0$ such that

$$P\{\rho_1 < 0\} > 0,\ P\{0 \le \rho_1 < a\} > 0 \text{ and } P\{\rho_1 \ge a\} > 0.$$

Let the interest rate be zero $r = 0$, i.e. $B_n \equiv B_0$ for all n and the stock evolution is given by $S_n = S_0 \prod_{i=1}^{n}(1 + \rho_i)$. Prove that there exist infinitely many martingale measures here.

2) Consider the following example of a game contingent claim in the CRR binomial market model where we assume that the horizon $N = 3$, the bond and the stock prices at time n are given by $B_n = 2^n$ and $S_n = \prod_{i=1}^{n}(1 + \rho_i)$, respectively, where $\rho_1, \rho_2, ...$ are i.i.d. random variables such that $\rho_i = 2$ with probability $1/4$ and $\rho_i = -\frac{1}{2}$ with probability $3/4$. The payoff function is given by

$$R(m, n) = Y_m \mathbb{I}_{m<n} + Z_n \mathbb{I}_{n \le m}$$

where $Y_n = Z_n + 1$ and $Z_n = \rho_n^2$. Find the fair price of this contingent claim and an optimal (rational) investment strategy of the seller and an optimal exercise time of the buyer.

Date: October 17, 2016.