Relative Topology and Connectedness

Let $S \subset \mathbb{R}^n$. and $p \in S$.
A **neighborhood of** p **relative to** S is a set $T \subset S$ that contains $B(p,r) \cap S$ for some $r > 0$.

$V \subset S$ is *open* relative to S if $V = S \cap U$ where U is an open set in $\mathbb{R}^n$.
The following are equivalent:

- F is closed relative to S.
- $F = S \cap K$, where K is closed in $\mathbb{R}^n$
- If x_j is a sequence of points in F that converge to a point $x \in S$, then $x \in F$.
- The complement of F relative to S, $S - F$, is open relative to S.

A set S is **disconnected** if there are two *disjoint* non-empty sets S_1 and S_2 such that $S = S_1 \cup S_2$ and both S_1 and S_2 are closed relative to S.

The only connected sets in $\mathbb{R}$ are intervals (possibly infinite):

$$a < x < b, \quad a \leq x \leq b, \quad a \leq x < b, \quad a < x \leq b$$

A function f is **continuous at a point** $p \in S$ if for every neighborhood V of $f(p)$, then the inverse image $f^{-1}(V)$ is a neighborhood of p relative to S.
In brief, if the inverse image of every open set is open.