**Math 425, Spring 2015** **Jerry L. Kazdan**

# Problem Set 3

DUE: Thurs. Feb. 5 in class. [Late papers will be accepted until 1:00 PM Friday.]

**This week**. Please read the first half of Chapter 3 in the Strauss text.

Most of the following problems are from the Strauss text. Lots of problems. Fortunately, many of them are short.

1. Problem p. 27 #1 was equivalent to showing that the solution to $u'' + cu = 0$ on $0 \le x \le 1$ with $u(0) = u(1) = 0$ may not be unique for certain values of $c > 0$. Consider the related problem

$$u'' - c(x)u = 0 \quad \text{where} \quad c(x) > 0. \tag{1}$$

   a) Show that there is no point $x_0$ in $0 < x < 1$ where $u(x_0) > 0$ and $u$ has a local maximum. Thus, $u$ cannot have a positive maximum at an interior point of this interval.
   b) Siimilarily, show that there is no point $x_0$ in $0 < x < 1$ where $u$ can have a negative local minimum.
   c) Conclude that if $u(0) = u(1) = 0$. then $u(x) = 0$ in the whole interval.
   MORAL: The sign of $c$ is important.

2. p. 31 # 1

3. p. 31 # 5

4. p. 38 # 8

5. p. 38 # 11

6. p. 41 # 4

7. p. 45 # 1

8. p. 46 # 4

9. To apply p. 46 #7a (which we proved in class) it is useful to know the explicit solution to some special cases of

$$u_t - u_{xx} = f(x,t) \quad \text{in} \quad 0 < x < L \quad \text{with} \quad u(0,t) = \phi(t),\ u(L,t) = \psi. \tag{2}$$

a) If $M$, $a$, and $b$ are constants, find the (unique) solution of

$$v_t - v_{xx} = M \quad \text{with} \quad v(0,t) = a \quad \text{and} \quad v(L) = b.$$

b) If $u(x,t)$ is a solution of equation (2) with $f \leq M$, $\phi \leq a$, and $\psi \leq b$, find an explicit function that gives an upper bound for $u$.

10. p. 52 # 1

11. p. 53 # 11

12. p. 54 # 16

[Last revised: March 16, 2015]